Subject:	Further Maths

Scheme title	Half term 1 - September	Half term 2 -October
Purpose of scheme	To develop fluency, problem solving and reasoning skills across the 6 key areas of number, algebra, geometry and measures, statistics, probability and ratio and proportion	To develop fluency, problem solving and reasoning skills across the 6 key areas of number, algebra, geometry and measures, statistics, probability and ratio and proportion
Skills	Algebra •Proof - Understand and use the structure of mathematical proof, proceeding from given assumptions through a series of logical steps to a conclusion; use methods of proof, including proof by deduction, proof by exhaustion. Disproof by counter example. •Algebraic expressions •Laws of indices - Understand and use the laws of indices for all rational exponents. •Surds - Use and manipulate surds, including rationalising the denominator. •Remainder and Factor Theorem •Algebraic division Quadratics and Cubics •Quadratic Equations – Manipulate polynomials algebraically, including expanding brackets and collecting like terms, factorisation and simple algebraic division; use of the factor theorem. •Quadratic Functions and Roots - Work with quadratic functions and their graphs; the discriminant of a quadratic function, including the conditions for real and repeated roots; completing the square; solution of quadratic equations including solving quadratic equations in a function of the unknown. •Quadratic Graphs - Understand and use graphs of functions; sketch curves defined by simple equations including polynomials, the modulus of a linear function, and (including their vertical and horizontal asymptotes); interpret algebraic solution of equations graphically; use intersection points of graphs to solve equations. •Understand and use proportional relationships and their graphs. •Factorising Cubics Inequalities and Simultaneous equations •Inequalities - Solve linear and quadratic inequalities in a single variable and interpret such inequalities graphically, including inequalities with brackets and fractions. Express solutions through correct use of 'and' and 'or', or through set notation. Represent linear and quadratic inequalities such as y > x + 1 and graphically. •Simultaneous Equations - Solve simultaneous equations in two variables by elimination and by substitution, including one linear and one quadratic equation. Co-ordinate geometry, Graphs and Circles •The Equation of a Straight Line - Understand and use the equation of a straight line, including the forms and ax + by + c = 0; gradient conditions for two straight lines to be parallel or perpendicular. •Be able to use straight line models in a variety of contexts. •Parallel and Perpendicular Lines and proportion •Curve Sketching and Graph Transformations - Understand the effect of simple transformations on the graph of y = f(x) including sketching associated graphs: y = af(x) , y = f(x) + a , y = f(x + a) , y = f(ax), •Circles - Understand and use the coordinate geometry of the circle including using the equation of a circle in the form ; completing the square to find the centre and radius of a circle; use of the following properties: he angle in a semicircle is a right angle •the perpendicular from the centre to a chord bisects the chord •the radius of a circle at a given point on its circumference is perpendicular to the tangent to the circle at that point. Further Maths Year 12Each teacher to teach simultaneously but in the order listed. Content is split in to Pure, Mechanics, Statistics and Discrete. A Level Maths content is indicated in black and A Level Further Maths content is indicated in blue. Trigonometry •The Sine and Cosine Rules - Understand and use the definitions of sine, cosine and tangent for all arguments; the sine and cosine rules; the area of a triangle in the form •Trig Identities - Understand and use Understand and use ; •Trig Graphs - Understand and use the sine, cosine and tangent functions; their graphs, symmetries and periodicity. •Solving Trig Equations - Solve simple trigonometric equations in a given interval, including quadratic equations in sin, cos and tan and equations involving multiples of the unknown angle. Exponentials and Logarithms •Exponentials - Know and use the function and its graph, where a is positive. . Know and use the function and its graph. . Know that the gradient of is equal to and hence understand why the exponential model is suitable in many applications. •Logarithms - Know and use the definition of as the inverse of , where a is positive and . Know and use the function Lnx and its graph. Know and use lnx as the inverse function of . •Laws of Logarithms - Understand and use the laws of logarithms: , •Solving Equations - Solve equations of the form •Modelling Exponential Growth and Decay - Understand and use exponential growth and decay; use in modelling (examples may include the use of e in continuous compound interest, radioactve decay, drug concentration decay, exponential growth as a model for population growth); consideration of limitations and refinements of exponential models •Using Logarithmic Graphs - Use logarithmic graphs to estimate parameters in relationships of the form and , given data for x and y, and . Vectors •Vectors - Use vectors in two dimensions •Calculating with Vectors - Calculate the magnitude and direction of a vector and convert between component form and magnitude/direction form. •Add vectors diagrammatically and perform the algebraic operations of vector addition and multiplication by scalars, and understand their geometrical interpretations. •Understand and use position vectors; calculate the distance between two points represented by position vectors. •Use vectors to solve problems in pure mathematics and in context, including forces •Modelling with Vectors	The Binomial Expansion • Binomial Expansions - Understand and use the binomial expansion of for positive integer n; the notations n!, nCr and ; link to binomial probabilities. Differentiation • The Gradient of a Curve - Understand and use the derivative of f(x) as the gradient of the tangent to the graph of y = f(x) at a general point (x, y); the gradient of the tangent as a limit; interpretation as a rate of change; sketching the gradient function for a given curve; second derivatives; differentiation from first principles for small positive integer powers of x • Differentiating y = f(x) – Differentiate , for rational values of n, and related constant multiples, sums and differences. • Second Order Derivatives - Understand and use the second derivative as the rate of change of gradient; • Derivatives of Graphs - Apply differentiation to find gradients, tangents and normals, maxima and minima and stationary points, • Identify where functions are increasing or decreasing. Real Life Problems Integration • Indefinite Integration - Know and use the Fundamental Theorem of Calculus. • Definite Integration - Integrate (excluding n = –1), and related sums, differences and constant multiples. • Evaluate definite integrals; use a definite integral to find the area under a curve Sampling, Data Presentation and Interpretation • Population Sampling - Understand and use the terms 'population' and 'sample'. Use samples to make infomal inferences about the population. • Understand and use sampling techniques, including simple random sampling and opportunity sampling. • Select or critique sampling techniques in the context of solving a statistical problem, including understanding that different samples can lead to different conclusions about the population. • LARGE DATA SET LESSON - Sampling • Representing Data - Interpret diagrams for single-variable data, including understanding that area in a histogram represents frequency. • Location: Mean, Median and Mode - Interpret measures of central tendency and variation, extending to standard deviation. Be able to calculate standard deviation, including from summary statistics. • LARGE DATA SET LESSON – Ungrouped/Grouped Data • Dispersion - Recognise and interpret possible outliers in data sets and statistical diagrams. • Select or critique data presentation techniques in the context of a statistical problem. • Be able to clean data, including dealing with missing data, errors and outliers. • LARGE DATA SET LESSON – Grouped Data • Correlation and Regression - Interpret scatter diagrams and regression lines for bivariate data, including recognition of scatter diagrams which include distinct sections of the population (calculations involving regression lines are excluded). • Understand informal interpretation of correlation. • Understand that correlation does not imply causation. • LARGE DATA SET LESSON – Time Series
	• Proof - Understand and use the structure of mathematical proof, proceeding from given assumptions through a series of logical steps to a conclusion; use methods of proof, including proof by deduction, proof by exhaustion. Disproof by counter example. • Algebraic expressions • Laws of indices - Understand and use the laws of indices for all rational exponents. • Surds - Use and manipulate surds, including rationalising the denominator. • Remainder and Factor Theorem • Algebraic division Quadratics and Cubics • Quadratic Equations – Manipulate polynomials algebraically, including expanding brackets and collecting like terms, factorisation and simple algebraic division; use of the factor theorem. • Quadratic Functions and Roots - Work with quadratic functions and their graphs; the discriminant of a quadratic function, including the conditions for real and repeated roots; completing the square; solution of quadratic equations including solving quadratic equations in a function of the unknown. • Quadratic Graphs - Understand and use graphs of functions; sketch curves defined by simple equations including polynomials, the modulus of a linear function, and (including their vertical and horizontal asymptotes); interpret algebraic solution of equations graphically; use intersection points of graphs to solve equations. • Understand and use proportional relationships and their graphs. • Factorising Cubics Inequalities and Simultaneous equations • Inequalities - Solve linear and quadratic inequalities in a single variable and interpret such inequalities graphically, including inequalities with brackets and fractions. 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Vectors • Vectors - Use vectors in two dimensions • Calculating with Vectors - Calculate the magnitude and direction of a vector and convert between component form and magnitude/direction form. • Add vectors diagrammatically and perform the algebraic operations of vector addition and multiplication by scalars, and understand their geometrical interpretations. • Understand and use position vectors; calculate the distance between two points represented by position vectors. • Use vectors to solve problems in pure mathematics and in context, including forces • Modelling with Vectors	
Key Words	Sine Cosine Trigonometry Symmetries Periodicity Modelling Exponential Logarithm Function Growth Decay Vector Magnitude Context Geometrical Modelling Forces DirectionSine Cosine Trigonometry Symmetries Periodicity Modelling Exponential Logarithm Function Growth Decay Vector Magnitude Context Geometrical Modelling Forces Direction+B5	Binomial Probability Gradient Rational Tangent Interpretation Decreasing Increasing Normals Maxima Minima Differential Integration Sampling Interpretation Single variable Correlation CausationKinematics Motion Displacement Velocity Speed Acceleration Derive Units Force Motion Models Mechanics Equilibrium Perpendicular
End Point	Students are able to understand and apply the skills identified above.	Students are able to understand and apply the skills identified above.
Assessment method	After each topic in bold (listed opposite), students complete a mini assessment. This may be done as part of home learning and sometimes done in class in test conditions. This is then teacher marked and recorded on the central tracking spreadsheet to inform progress and intervention. Students complete full A level assessments in line with the AQA specification at progress points in the year in line with the school calendar. Assessments are cumulative and grade boundaries reflect actual A Level Maths grade boundaries	After each topic in bold (listed opposite), students complete a mini assessment. This may be done as part of home learning and sometimes done in class in test conditions. This is then teacher marked and recorded on the central tracking spreadsheet to inform progress and intervention. Students complete full A level assessments in line with the AQA specification at progress points in the year in line with the school calendar. Assessments are cumulative and grade boundaries reflect actual A Level Maths grade boundaries

Year Group:	12

Half term 3 - January	Half term 4 - February	Half term 5 - April
To develop fluency, problem solving and reasoning skills across the 6 key areas of number, algebra, geometry and measures, statistics, probability and ratio and proportion	To develop fluency, problem solving and reasoning skills across the 6 key areas of number, algebra, geometry and measures, statistics, probability and ratio and proportion	To develop fluency, problem solving and reasoning skills across the 6 key areas of number, algebra, geometry and measures, statistics, probability and ratio and proportion
Probability •Elementary Probability - Understand and use mutually exclusive and independent events when calculating probabilities. •Link to discrete and continuous distributions. •Solving Probability Problems •Laws of Probability Statistical Distributions •Probability Distributions - Understand and use simple, discrete probability distributions (calculation of mean and variance of discrete random variables is excluded), including the binomial distribution, as a model; calculate probabilities using the binomial distribution. •Binomial Distributions •The Binomial Cumulative •Modelling Real Problems Statistical Hypothesis Testing •Hypothesis Testing - Understand and apply the language of statistical hypothesis testing, developed through a binomial model: null hypothesis, alternative hypothesis, significance level, test statistic, 1-tail test, 2-tail test, critical value, critical region, acceptance region, p-value; •Hypothesis Tests for Binomial Distribution - Conduct a statistical hypothesis test for the proportion in the binomial distribution and interpret the results in context. •Understand that a sample is being used to make an inference about the population and appreciate that the significance level is the probability of incorrectly rejecting the null hypothesis. Further Maths Year 12Each teacher to teach simultaneously but in the order listed. Content is split in to Pure, Mechanics, Statistics and Discrete. A Level Maths content is indicated in black and A Level Further Maths content is indicated in blue. Complex Numbers 1 •Properties and arithmetic - Add, subtract, multiply and divide complex numbers in the form x + iy with x and y real; understand and use the terms 'real part' and 'imaginary part'. •Understand and use the complex conjugate; know that non-real roots of polynomial equations with real coefficients occur in conjugate pairs. •Solving polynomial equations - Solve any quadratic equation with real coefficients; solve cubic or quartic equations with real coefficients (given sufficient information to deduce at least one root for cubics or at least one complex root or quadratic factor for quartics). •Argand diagrams - Use and interpret Argand diagrams. •Modulus-argument form and loci - Convert between the Cartesian form and the modulus-argument form of a complex number (knowledge of radians is assumed). •Multiply and divide complex numbers in modulus-argument form (knowledge of radians and compound angle formulae is assumed). •Construct and interpret simple loci in the Argand diagram such as and (knowledge of radians is assumed). Curve Sketching 1 •Linear rational functions - Graphs of rational functions of form , asymptotes, points of intersection with coordinate axes or other straight lines; associated inequalities. •Graphs of rational functions of form including cases when some of these coefficients are zero; asymptotes parallel to coordinate axes; •Quadratic rational functions - Using quadratic theory (not calculus) to find the possible values of the function and coordinates of the stationary points of the graph for rational functions of form •Sketching graphs of curves with equation including intercepts with axes and equations of asymptotes of hyperbolas. •Polar coordinates - Understand and use polar coordinates and be able to convert between polar and Cartesian coordinates. •Sketch curves with r given as a function of , including use of trigonometric functions. θ •π •Parabolas, ellipses and hyperbolas •Hyperbolic functions 1 - Understand the definitions of hyperbolic functions sinh x, cosh x and tanh x, and be able to sketch their graphs. Understand and be able to use the definitions of the inverse hyperbolic functions Derive and use the logarithmic forms of the inverse hyperbolic functions. •Understand and use ,	Graphs and Networks 1 •Graphs and networks - Understand and use the language of graphs including: vertex, edge, trail, cycle, connected, degree, subgraph, subdivision, multiple edge and loop. •Identify or prove properties of a graph including that a graph is Eulerian, semi-Eulerian or Hamiltonian. •Understand and use Euler's formula for connected planar graphs. •Understand and use complete graphs and bipartite graphs, including adjacency matrices and the complement of a graph. •Understand and use simple graphs, simple-connected graphs and trees. •Traversing a graph - Understand and use the language of networks including: node, arc and weight. •Minimum spanning trees and Kruskal's algorithm - Solve network optimisation problems using spanning trees. •Minimum spanning trees and Prim's algorithm •The route inspection problem - Solve route inspection problems. •The travelling salesperson problem - Find and interpret upper bounds and lower bounds for the travelling salesperson problem. •Network flows 1 - Evaluate, modify and refine models which use networks. •Interpret flow problems represented by a network of directed arcs. •Find the value of a cut and understand its meaning. •Use and interpret the maximum flow-minimum cut theorem. •Introduce supersources and supersinks to a network. Critical Path Analysis 1 •Activity networks - Construct, represent and interpret a precedence (activity) network using activity-on-node •Determine earliest and latest start and finish times for an activity network. •Critical paths - Identify critical activities, critical paths and the float of non-critical activities. •Refine models and understand the implications of possible changes in the context of critical path analysis. Linear Programming and Game Theory •Constrained optimization - Formulate constrained optimisation problems. •Solve constrained optimisation problems via graphical methods. •Zero sum games - Understand, interpret and construct pay-off matrices. •Find play-safe strategies and the value of the game. •Prove the existence or non-existence of a stable solution. •Identify and make use of dominated strategies. •Mixed strategy games - Find optimal mixed strategies for a game including use of graphical methods. Further Maths Year 12Each teacher to teach simultaneously but in the order listed. Content is split in to Pure, Mechanics, Statistics and Discrete. A Level Maths content is indicated in black and A Level Further Maths content is indicated in blue. Matrices 1 •Properties and arithmetic - Add, subtract and multiply conformable matrices; multiply a matrix by a scalar. •Understand and use zero and identity matrices. Construct proofs using mathematical induction; contexts include sums of series, divisibility, and powers of matrices. •Calculate determinants of matrices 2×2 . •Understand and use Singular and non-singular matrices; properties of inverse matrices. Calculate and use the inverse of non-singular matrices 2×2 •Transformations - Use matrices to represent linear transformations in 2D; successive transformations; single transformations in 3D (3D transformations confined to reflection in one of x = 0, y = 0, z = 0 or rotation about one of the coordinate axes) (knowledge of 3D vectors is assumed). •Systems of linear equations - Find invariant points and lines for a linear transformation. Vectors 1 •The vector equation of a line - Understand and use the vector and Cartesian forms of an equation of a straight line in 3D. Calculate the scalar product and use it to calculate the angle between two lines. •Find the intersection of two lines. Calculate the perpendicular distance between two lines, from a point to a line •The scalar product - Check whether vectors are perpendicular by using the scalar product. Forces and Energy •Work, energy and power - Work done by a force acting in the direction of motion or directly opposing the motion. Use of . WD=Fscosθ •Gravitational potential energy. Use in conservation of energy problems. Kinetic energy. Use in conservation of energy problems. •Finding dimensions of quantities; checking for dimensional consistency. •Prediction of formulae; finding powers in potential formulae. •Hooke's law - including using modulus of elasticity. •Work done by a variable force. Use of . WD= ∫Fdx Use in conservation of energy problems. Work done by a force acting in the direction of motion or directly opposing the motion. •Elastic potential energy using modulus of elasticity. Use of •Power (resolving will not be required at AS level, problems which require resolving will be required at A-level). Use of P = Fv.	Abstract Algebra •Binary Operations - Understand and use binary operations including use of modular arithmetic and matrix multiplication. •Understand, use and prove the commutativity of a binary operation. •Understand, use and prove the associativity of a binary operation. •Construct a Cayley table for a given set under a given binary operation. •Understand and prove the existence of an identity element for a given set under a given binary operation. •Find the inverse of an element belonging to a given set under a given binary operation. •Modular arithmetic Algebra and Series •Roots of polynomials - Understand and use the relationship between roots and coefficients of polynomial equations up to quartic equations. •Form a polynomial equation whose roots are a linear transformation of the roots of a given polynomial equation (of at least cubic degree). •Understand and use formulae for the sums of integers, squares and cubes and use these to sum other series. •Inequalities - Involving polynomial equations (cubic and quartic). •Solve inequalities such as algebraically •Summing series and the method of differences - Understand and use the method of differences for summation of series •Single transformations of curves involving translations, stretches parallel to coordinate axes and reflections in the coordinate axes and the lines . y=±x •Proof by induction •Maclaurin series 1 - Recognise and use the Maclaurin series for , , , , and be aware of the range of values of x for which they are valid (proof not required). Integration 1 •Mean values - Understand and evaluate the mean value of a function. •Volume of revolution - Derive formulae for and calculate volumes of revolution. Further Maths Year 12 Teacher 2:Half Term 5 - April Each teacher to teach simultaneously but in the order listed. Content is split in to Pure, Mechanics, Statistics and Discrete. A Level Maths content is indicated in black and A Level Further Maths content is indicated in blue. Momentum •Conservation of momentum - for linear motion and cases where velocities are given as one or two dimensional vectors (resolving will not be required at AS level, •Collisions- Coefficient of restitution and Newton's Experimental Law. Use in direct collisions and impacts with a fixed smooth surface •Impulses - Impulse and its relation to momentum (in one- or two-dimensions) (resolving will not be required at AS level. Use of Ft = mv – mu •Impulse for variable forces. One dimension only. Use of I=∫ Fdt Circular Motion •Kinematics of circular motion - Motion of a particle moving in a circle with constant speed (knowledge of radians assumed). •Understand the definition of angular speed. •Use both radians and revolutions per unit time. •Relationships between speed, angular speed, radius and acceleration. Use of , v=rω, •Horizontal circular motion
Probability Independent Discrete Continuous Laws Probability Cumulative Modelling Hypothesis Critical value Critical region Binomial Significance Null hypothesis Alternate hypothesis Properties Arithmetic Complex Conjugate Quartic Polynomial Cubic Cartesian Modulus-argument Radians Linear Rational Inequalities Theory Curves Function Hyperbolic Ellipse Parabola Hyperbola	Vertex Edge Trail Cycle Connected Degree Subgraph Subdivision Loop Eulerian Semi-eulerian Hamiltonian Complete graph Adjacency matrix Kruskal Prim Supersource Supersink Model DominanceProperties Arithmetic Matrices Powers Linear Inverse Vector Scalar Work Energy Power Gravitational potential energy Conservation Dimensions Formulae Variable Modulus Elasticity Power	Binary Cayley Operation Modular arithmetic Polynomials Inequalities Curves Induction Maclaurin series Commutativity Associativity Inequalities Proof Mean values Method of differences Transformations Cubic DegreeMomentum Conservation Velocities Dimensional Coefficient Newton Collisions Momentums Surface Relation Kinematics Circular Angular speed Radians Revolutions Horizontal Vertical
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