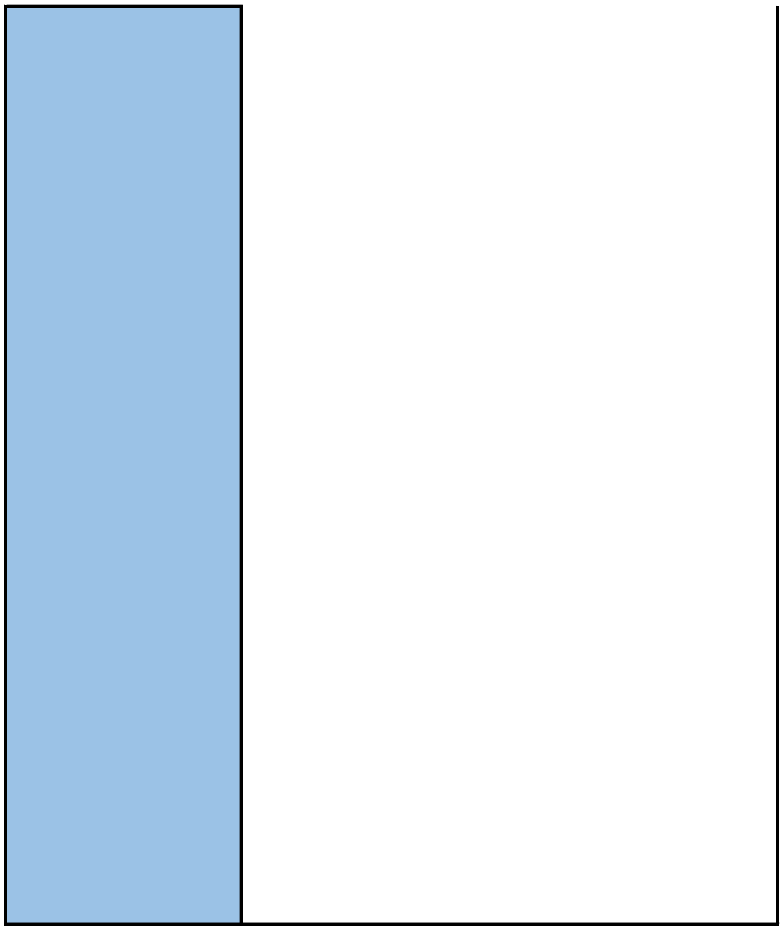
Subject :	Further Maths

Scheme title	Half term 1 - June
Purpose of scheme	To develop fluency, problem solving and reasoning skills across the 6 key areas of number, algebra, geometry and measures, statistics, probability and ratio and proportion
Skills	Teacher 1: Algebra and Functions •Proof by Contradiction - (including proof of the irrationality of √2 and the infinity of primes, and application to unfamiliar proofs). •Simplifying Expressions - Simplify rational expressions including by factorising and cancelling, and algebraic division (by linear expressions only). •Mapping and Functions - Use of functions in modelling, including consideration of limitations and refinements of the models. •Composite/Inverse Functions - Understand and use composite functions; inverse functions and their graphs. •Modulus - the modulus of a linear function •Transformation of Graphs •Partial Fractions - Decompose rational functions into partial fractions (denominators not more complicated than squared linear terms and with no more than three terms, numerators constant or linear). Teacher 2: Sequences and Series •Sequences - Work with sequences including those given by a formula for the nth term and those generated by a simple relation of the form $x_{n+1} = f(x_n)$; increasing sequences; decreasing sequences; periodic sequences. •Arithmetic Sequences and series - Understand and work with arithmetic sequences and series, including the formulae for nth term and the sum to n terms. •Geometric Sequences and Series - Understand and work with geometric sequences and series including the formulae for the nth term and the sum of a finite geometric series; the sum to infinity of a convergent geometric series, including the use of $\|r\| < 1$; modulus notation. •Modelling Problems - Use sequences and series in modelling.



Key Words	Proof by contradiction Simplify Expressions Factorise Cancel Mappings Functions Inverse Linear Modulus Partial fractions Denominators Linear Sequences Arithmetic Geometric Modelling
End Point	Students are able to understand and apply the skills identified above.
Assessment method	After each topic in bold (listed opposite), students complete a mini assessment. This may be done as part of home learning and sometimes done in class in test conditions. This is then teacher marked and recorded on the central tracking spreadsheet to inform progress and intervention. Students complete full A level assessments in line with the AQA specification at progress points in the year in line with the school calendar. Assessments are cumulative and grade boundaries reflect actual A Level Maths grade boundaries.

Half term 2 - September
To develop fluency, problem solving and reasoning skills across the 6 key areas of number, algebra, geometry and measures, statistics, probability and ratio and proportion
Parametric Equations Parametric Equations of Curves - Understand and use the parametric equations of curves and conversion between Cartesian and parametric forms. Parametric and Cartesian Equations - Use parametric equations in modelling in a variety of contexts. Differentiation 2 Points of inflection - connection to convex and concave sections of curves and points of inflection. Chain Rule, Product Rule, Quotient Rule - Differentiate using the product rule, the quotient rule and the chain rule, including problems involving connected rates of change and inverse functions. Differentiation of e, ln x and a^x - and related sums, differences and constant multiples. Understand and use the derivative of ln x. Differentiating Trig Functions - differentiation from first principles for small positive integer powers of x and for and sinx and cosx Connected Rates of Change - Construct simple differential equations in pure mathematics and in context, (contexts may include kinematics, population growth and modelling the relationship between price and demand). Differentiation with Parametric Equations and implicit differentiation - Differentiate simple functions and relations defined implicitly or parametrically, for first derivative only. Integration Integration of ⟦(ax+b)⟧^n Integration e^x and 1/x, sinkx, coskx and related sums, differences and constant multiples. Integration of Trigonometric Functions Integration of (f'(x))/(f(x)) Integrating du/dx f’(u) Using Trig Identities in Integration Finding Area – including the area between two curves. Parametric Integration Integration by Substitution - Carry out simple cases of integration by substitution and integration by parts; understand these methods as the inverse processes of the chain and product rules respectively. (Integration by substitution includes finding a suitable substitution and is limited to cases where one substitution will lead to a function which can be integrated; integration by parts includes more than one application of the method but excludes reduction formulae.) Integration by Parts Integration Using Partial Fractions - Integrate using partial fractions that are linear in the denominator. Differential Equations - Evaluate the analytical solution of simple first order differential equations with separable variables, including finding particular solutions. (Separation of variables may require factorisation involving a common factor.)

Understand and use integration as the limit of a sum - Interpret the solution of a differential equation in the context of solving a problem, including identifying limitations of the solution; includes links to kinematics.

Further Maths Year 13Each teacher to teach simultaneously but in the order listed. Content is split in to Pure, Mechanics, Statistics and Discrete.
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The Binomial Expansion 2
The Binomial Expansion - Extend to any rational n, including its use for approximation; be aware that the expansion is valid for (Proof not required.)
Binomial Expansion by Approximations
Binomial Expansion and Partial Fractions

Trigonometry 2
Arcs and Sectors - Work with radian measure, including use for arc length and area of sector.
Know and use exact values of sin, tan and cos for and multiples thereof.
Small Angle Approximations - Understand and use the standard small angle approximations of sine, cosine and tangent

Inverse Trig Functions - Understand and use the definitions of secant, cosecant and cotangent and of arcsin, arccos and arctan; their relationships to sine, cosine and tangent; understanding of their graphs; their ranges and domains.
Cosec, Sec and Cot
Identities involving Cosec, Sec and Cot – understand and use   Construct proofs involving trigonometric functions and identities.
The additional Formulas - Understand and use double angle formulae; use of formulae for and
The Double Angle Formulas
The R Addition Formulas - Understand and use expressions for in the equivalent forms of or
Modelling with Trig Functions - Use trigonometric functions to solve problems in context, including problems involving vectors, kinematics and forces.
Vectors
Vectors in Three Dimensions
Calculating with Vectors - Use vectors to solve problems in pure mathematics and in context, including forces and kinematics.

Parametric
Cartesian
Product
Quotient
Chain rule
Trigonometrically
Integration
Integral
Partial fraction
Limitation
Sum
Kinematics
Analytical
Particular solutions
Interpret
MethodBinomial
Approximation
Partial fraction
Radian
Arc
Sector
Trigonometric
Relationships
Inverse trigonometry
Equivalent
Kinematics
Forces
Vectors
Forces
Proof
Range
Domain

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Half term 3 - October
To develop fluency, problem solving and reasoning skills across the 6 key areas of number, algebra, geometry and measures, statistics, probability and ratio and proportion
Numerical Methods •Location of Roots - Locate roots of f(x)=0 by considering changes of sign f(x) of in an interval of x on which f(x) is sufficiently well-behaved. •Understand how change of sign methods can fail. •Iterative Methods - Solve equations approximately using simple iterative methods; be able to draw associated cobweb and staircase diagrams. The Newton-Raphson Method - Solve equations using the Newton-Raphson method and other recurrence relations of the form Understand how such methods can fail. •The Trapezium Rule - Understand and use numerical integration of functions, including the use of the trapezium rule and estimating the approximate area under a curve and limits that it must lie between. •Use numerical methods to solve problems in context. Correlation and Regression •Regression •The Product Moment Correlation Coefficient •Rank Correlations •LARGE DATA SET LESSON – Correlation Probability 2 •Conditional Probability - Understand and use conditional probability, including the use of tree diagrams, Venn diagrams, two-way tables. •Understand and use the conditional probability formula •Modelling with Probability - including critiquing assumptions made and the likely effect of more realistic assumptions. The Normal Distribution •The Normal Distribution - Understand and use the Normal distribution as a model; find probabilities using the Normal distribution. •Link to histograms, mean, standard deviation, points of inflection and the binomial distribution. •Normal Approximation •Choosing Probability Distributions - Select an appropriate probability distribution for a context, with appropriate reasoning, including recognising when the binomial or Normal model may not be appropriate. •Hypothesis Tests - Understand and apply the language of statistical hypothesis testing, developed through a binomial model: null hypothesis, alternative hypothesis, significance level, test statistic, 1-tail test, 2-tail test, critical value, critical region, acceptance region, p-value; extend to correlation coefficients as measures of how close data points lie to a straight line and be able to interpret a given correlation coefficient using a given p-value or critical value (calculation of correlation coefficients is excluded). •Conduct a statistical hypothesis test for the mean of a Normal distribution with known, given or assumed

variance and interpret the results in context.
•LARGE DATA SET LESSON – Hypothesis Testing

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Kinematics 2
•Projectiles - Understand, use and derive the formulae for constant acceleration for motion in a straight line; extend to two dimensions using vectors.
•Use calculus in kinematics for motion in a straight line: extend to two dimensions using vectors.
•Non-Uniform Acceleration in 2 dimensions - Model motion under gravity in a vertical plane using vectors; projectiles.

Dynamics
•Resolving Forces - Understand and use addition of forces; resultant forces; dynamics for motion in a plane.
•Friction - Understand and use the model for friction; coefficient of friction; motion of a body on a rough surface; limiting friction and statics.
•Newton's Law of Motion - Understand and use Newton's second law for motion in a straight line (restricted to forces in two perpendicular directions or simple cases of forces given as 2D vectors); extend to situations where forces need to be resolved (restricted to 2 dimensions).
•Understand and use Newton's third law; equilibrium of forces on a particle and motion in a straight line (restricted to forces in two perpendicular directions or simple cases of forces given as 2D vectors); application to problems involving smooth pulleys and connected particles; resolving forces in two dimensions; equilibrium of a particle under coplanar forces.

Moments
•Moments - Understand and use moments in simple static contexts.
•Reaction Forces and Friction in Moments

Roots
Interval
Approximate
Newton-Raphson
Trapezium
Context
Regression
Correlation
Rank
Conditional
Assumptions
Critiquing
Normal distribution
Null hypothesis
Alternate hypothesis
Standard deviation
Variance
CoefiicientsProjectile
Dimension
Model
Gravity
Kinematics
Resolving
Friction
Motion
Equilibrium
Coplanar
Forces
Vectors
Moment
Friction
Reaction forces
Friction
Force

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Year Group:

Half term 4 - January
To develop fluency, problem solving and reasoning skills across the 6 key areas of number, algebra, geometry and measures, statistics, probability and ratio and proportion
Graphs and Networks 2 •Planar graphs and isomorphisms - Use Kuratowski's Theorem to determine the planarity of graphs. •Recognise and find isomorphism between graphs. •Network flows 2 - Augment flows and determine the maximum flow in a network. •Solve problems involving arcs with upper and lower capacities. •Refine network flow problems including using nodes of restricted capacity. Critical Path Analysis 2 •Gantt charts - Construct and interpret Gantt (cascade) diagrams and resource histograms. •Resourcing - Carry out resource levelling (using heuristic procedures) and solve problems where resources are restricted. Linear Programming and Game Theory 2 •Simplex algorithm - Use the Simplex algorithm for optimising (maximising and minimising) an objective function including the use of slack variables. •Interpret a Simplex tableau. •Games as linear programming problems - Convert and solve higher order games to linear programming problems. Group Theory •Groups - Understand and use the language of groups including: order, period, subgroup, proper, trivial, non-trivial. •Understand and use the group axioms: closure, identity, inverses and associativity, including use of Cayley tables. •Recognise and use finite and infinite groups and their subgroups, including: groups of symmetries of regular polygons, cyclic groups and abelian groups. •Understand and use Lagrange's theorem. •Identify and use the generators of a group. •Isomorphisms - Recognise and find isomorphism between groups of finite order. Series •Summing series using partial fractions - Understand and use the method of differences for summation of series including use of partial fractions. •Maclaurin Series 2 - Find the Maclaurin series of a function including the general term. •Evaluation of limits using Maclaurin series or l'Hôpital's rule. Further Maths Year 13Each teacher to teach simultaneously but in the order listed. Content is split in to Pure, Mechanics, Statistics and Discrete. A Level Maths content is indicated in black and A Level Further Maths content is indicated in blue. Complex Numbers 2 •Exponential form - Know and use the definition •De Moivre's theorem - Understand de Moivre's theorem and use it to find multiple angle formulae and sums of series.

- Roots of unity - Find the n distinct nth roots of and know that they form the vertices of a regular n-gon in the Argand diagram.
- Use complex roots of unity to solve geometric problems.

Curve Sketching 2
- Reciprocal, modulus graphs and associated inequalities.
- Transformations - Single transformations of curves involving translations, stretches parallel to coordinate axes and reflections in the coordinate axes and the lines . y=±x
- Extend to composite transformations including rotations and enlargements.
- Hyperbolic functions 2
- Rational functions with oblique asymptotes

Differential Equations
- First order differential equations - Find and use an integrating factor to solve differential equations of the form and recognise when it is appropriate to do so. Find both general and particular solutions of differential equations.
- Second order equations - Solve differential equations of the form where a and b are constants, by using the auxiliary equation.
- Solve differential equations of the form where a and b are constants by solving the homogeneous case and adding a particular integral to the complementary function (in cases where f(x) is a polynomial, exponential or trigonometric function).
- Understand and use the relationship between the cases when the discriminant of the auxiliary equation is positive, zero and negative and the form of solution of the differential equation.
- Simple harmonic motion - Solve the equation for simple harmonic motion and relate the solution to the motion.
- Damped and forced harmonic motion - Model damped oscillations using 2nd order differential equations and interpret the solutions. Use models for damped motion where the damping force is proportional to the velocity.
- Understand light, critical and heavy damping and be able to determine when each will occur.
- Coupled equations - Use differential equations in modelling in kinematics and in other contexts.
- Analyse and interpret models of situations with one independent variable and two dependent variables as a pair of coupled 1st order simultaneous equations and be able to solve them, for example predator-prey models.
- Use of Hooke's Law with T = kx to formulate a differential equation for simple harmonic motion, where k is a constant.

Isomorphism
Augment flows
Capacity
Gantt chart
Resourcing
Simplex algorithm
Groups
Axiom
Subgroup
Trivial
Cyclic
Abelian group
Cayley table
Summing
Series
Limit
FunctionComplex
Roots
Reciprocal
Inequalities
Modulus
Asymptotes
Complementary function
Harmonic motion
Auxillary equation
Differential equation
Particular integral
Complementary function
Interpret
Hooke's law
Simultaneous
Independent
Variable

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Half term 5 - February
To develop fluency, problem solving and reasoning skills across the 6 key areas of number, algebra, geometry and measures, statistics, probability and ratio and proportion
Integration 2 •Integrate using partial fractions (extend to quadratic factors in the denominator). axc+2 •Improper integrals - Evaluate improper integrals where either the integrand is undefined at a value in the range of integration or the range of integration extends to infinity. •Inverse trigonometric functions - Differentiate inverse trigonometric functions. •Integrate functions of the form and be able to choose trigonometric substitutions to integrate associated functions. •Hyperbolic functions - Understand the definitions of hyperbolic functions sinh x, cosh x and tanh x, including their domains and ranges, and be able to sketch their graphs. •Understand the definitions of hyperbolic functions sech x, cosech x and coth x, including their domains and ranges. •Differentiate and integrate hyperbolic functions. •Understand and be able to use the definitions of the inverse hyperbolic functions and their domains and ranges. • •Construct proofs involving hyperbolic functions and identities. Partial fractions •Integrate using partial fractions (extend to quadratic factors in the denominator). •Reduction formulae - Derivation and use of reduction formulae for integration. •Polar graphs and areas - Find the area enclosed by a polar curve. •Lengths and surface areas - Arc length and area of surface of revolution for curves expressed in Cartesian or parametric coordinates. Numerical Methods •Numerical integration - Mid-ordinate rule and Simpson's rule for integration. •Euler's method - Euler's step by step method for solving first order differential equations. •Improved Euler method for solving first order differential equations. •The limits applied to improper integrals. Matrices 2 •Determinants, inverse matrices and linear equations - Calculate determinants of matrices and matrices and interpret as scale factors, including the effect on orientation. 2×2, 3×3 •Understand and use singular and non-singular matrices; properties of inverse matrices. •Calculate and use the inverse of non-singular matrices and matrices. 2×2 3×3 •Solve three linear simultaneous equations in three variables by use of the inverse matrix. •Interpret geometrically the solution and failure of solution of three simultaneous linear equations. •Manipulating determinants - Factorisation of determinants using row and column operations. •Eigenvalues and eigenvectors - Find eigenvalues and eigenvectors of and matrices. 2×2 3×3 •Find and use the characteristic equation. •Understand the geometrical significance of eigenvalues and eigenvectors. •Diagonalisation of matrices; when eigenvalues are real. Further Maths Year 13Each teacher to teach simultaneously but in the order listed. Content is split in to Pure, Mechanics, Statistics and Discrete.

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Vectors 2

- The vector product - Understand and use the vector and Cartesian forms of the equation of a plane.
- Calculate the scalar product and use it to calculate the angle between two lines, to express the equation of a plane, and to calculate the angle between two planes and the angle between a line and a plane.
- Calculate and understand the properties of the vector product.
- Understand and use the equation of a straight line in the form .
- Use vector products to find the area of a triangle.
- The equation of a plane - Find the intersection of a line and a plane.
- Finding distances 2 - Calculate the perpendicular distance between two lines, from a point to a line and from a point to a plane.

Circular Motion 2

- Kinematics of circular motion 2 - Use position, velocity and acceleration as vectors in the context of circular motion.
- The conical pendulum - Conical pendulum, with one or two strings.
- Vertical circular motion - Circular motion in a vertical plane. Includes conditions to complete vertical circles. Use conservation of energy in this context.

Centres of Mass and Stability

- Moments and couples - Conservation of momentum for linear motion and cases where velocities are given as one or two dimensional vectors (resolving will not be required at AS level, problems which require resolving will be required at A-level).
- Centre of mass for point masses and laminas - Coefficient of restitution and Newton's Experimental Law. Use in direct collisions and impacts with a fixed smooth surface (resolving will not be required at AS level, problems which require resolving will be required at A-level).
- Impulse and its relation to momentum (in one- or two-dimensions) (resolving will not be required at AS level, problems which require resolving will be required at A-level).
- Use of Ft = mv – mu (resolving will not be required at AS level, problems which require resolving will be required at A-level).
- Centre of mass for laminas and solids:-
- Centre of mass for a system of particles.
- Centre of mass for a composite body.
- Centre of mass of a lamina by integration.
- Centres of mass of bodies formed by rotating a region about the x-axis.
- Conditions for sliding and toppling. Problems including suspension and on an inclined plane.
- Equilibrium - Determine the forces acting on a rigid body in equilibrium. Use of moments and couples.

Integration
Inverse
Hyperbolic
Proof
Identity
Partial fraction
Evaluate
Integrate
Derivation
Polar curve
Parametric
Limits
Improper integral
Determinant
Matrix
Diagonalization
Eigenvalues
Eigenvectors
GeometricallyVector
Cartesian
Plane
Product
Perpendicular
Intersection
Kinematics
Conical
Pendulum
Circular motion
Energy
Moments
Couples
Momentum
Collision
Coefficient of restitution

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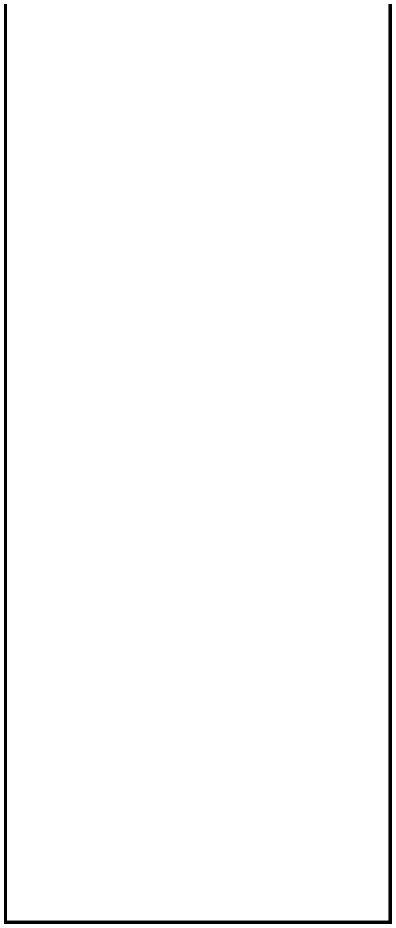
Half term 6 - April
To develop fluency, problem solving and reasoning skills across the 6 key areas of number, algebra, geometry and measures, statistics, probability and ratio and proportion
Teacher 1 and Teacher 2 Complete any content not finished from HT5 then revision of content from A Level Maths and A Level Further Maths including resources such as: •Alpha Books 1-6 •Integral Revision and Summary Sheets: https://integralmaths.org/ This has a wealth of resources for both maths and further maths including student access to videos. Please see our website list for the latest log-in details. •Practice Exam Papers from AQA and CGP •Materials from the Resource Bank on T drive: T:\bec\Curriculum.Enrichment.Inclusion\Curriculum\Maths and Computer Science\Maths\Curriculum and SOW NEW\Sixth Form\RESOURCE BANK •This contains a wealth of resources, please keep adding to this. •Websites such as Maths and Physics Tutor, Mr Barton and Integral Maths •Make use of the CGP A Level Maths Revision Guide available from SFO •Use of knowledge organisers



Key Vocabulary is outlined throughout the SOW
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