Solving Problems

1 **If you're not sure what to do, try something!**
When you are solving a problem, it may not be immediately obvious what you should do. Don't be afraid to try something just because you think it may not work! Often, trying something that doesn't work helps you to understand the problem better, and leads you to a more useful approach.

2 **Check any formula you have found**
When you find a mathematical expression to describe a particular situation, make sure that it is correct for a simple example.

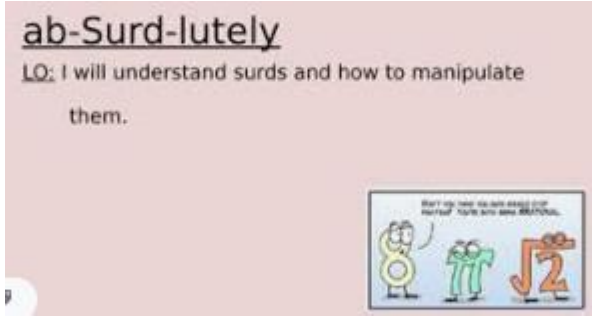
3 **Think about assumptions in modelling**
When modelling a real life situation, always think about the assumptions that you are making, and whether they are realistic.

Notation and Proof

1 **Be careful with notation**
The force in this equation is the driving force of the engine only.

2 **Think carefully about the meaning of mathematical statements**
Remember that if a statement is true, this does not necessarily mean that its converse is true. A => B does not mean that B <= A. If the converse is true, then you can write A⇔B.

3 **Make sure that a proof really is a proof**
Remember that to prove a result, you must show that it is true in all possible cases. If it is not possible to test all cases, then you need to generalise



Surds

1 **Make sure you can write surds in their simplest form**
Try to get into the habit of writing surds in terms of the simplest surd possible whenever you can – it usually makes them easier to work with.
e.g. write 8 as 2√2
write 12 as 2√3

2 **Be careful when multiplying surds**
Remember that when you are multiplying two expressions involving surds together, you must use brackets and treat them in the same way as you would an algebraic expression –each term in one bracket must be multiplied by each term in the other.

✗ *Wrong*
$$(1+\sqrt{2})(3-\sqrt{2}) = 3-(\sqrt{2})^2 = 3-2 = 1 \quad ✗$$

✓ *Right*
$$(1+\sqrt{2})(3-\sqrt{2}) = 3-\sqrt{2}+3\sqrt{2}-(\sqrt{2})^2 = 3+2\sqrt{2}-2 = 1+2\sqrt{2} \quad ✓$$

3 **Take care when rationalising the denominator**
Remember that when you are rationalising a denominator you must multiply top and bottom by the same expression.

✗ *Wrong*
$$\frac{2+\sqrt{3}}{1-\sqrt{2}} = \frac{(2+\sqrt{3})(2-\sqrt{3})}{(1-\sqrt{2})(1+\sqrt{2})} \quad ✗$$

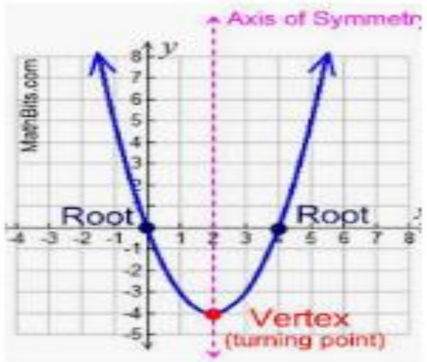
✓ *Right*
$$\frac{2+\sqrt{3}}{1-\sqrt{2}} = \frac{(2+\sqrt{3})(1+\sqrt{2})}{(1-\sqrt{2})(1+\sqrt{2})} \quad ✓$$

Indices

1	**Make sure you use the law of indices in appropriate situations** Remember you cannot apply the laws of indices to the sum or difference of two expressions involving indices (although you may be able to simplify in another way.) ✘ Wrong $a^2 + a^5 = a^7$ ✘ ✓ Right $a^2 + a^5 = a^2(1 + a^3)$ ✓
2	**Look at the base** Make sure that you only apply the first two laws of indices to expressions with the same base ✘ Wrong $2^2 \times 3^5 = 6^7$ ✘ ✓ Right $2^2 \times 2^5 = 2^7$ ✓
3	**Remember the value of a^0** a^0 is always 1, for any value of a

Quadratic Graphs & Equations

1	**Make sure that you can multiply out and factorise confidently** These algebraic skills are vital in many areas of mathematics at this level. Practise them until you can do them confidently.
2	**Remember the relationship between the solutions of a quadratic equation and the corresponding quadratic graph** The solutions of a quadratic equation tell you where the corresponding quadratic graph crosses the x-axis. This is very useful in sketching quadratic graphs.
3	**Use completing the square to find the turning point (vertex)** Remember that you can find the maximum or minimum point of a quadratic from its completed square form.
4	**Recognise the nature of the vertex** Make sure that you know how to tell whether a quadratic graph has a maximum or a minimum point.



Quadratic Graphs & Equations

5	**Be careful when the coefficient of x^2 is not 1** When completing the square for a quadratic with a coefficient of x^2 which is not 1, make sure that you take out the coefficient of x^2 as a factor first.
6	**When completing the square, be careful with signs** Be especially careful when dealing with expressions where the coefficient of x^2 is negative.
7	**Check your answers** After completing the square, you can always check by multiplying out and making sure that you get the original quadratic expression.
8	**When solving problems, make sure your answer makes sense** Always look at your answers to problems in the light of the original question. If there are two solutions, do they both make sense, or should one be discarded? Think carefully about the meaning of a negative solution, since this may or may not be a valid solution.

The Quadratic Formula

1	**Remember how to calculate the discriminant and what information it gives you** Remember that the discriminant of a quadratic equation gives you useful information about the nature of its solutions, so it is often useful to work out the discriminant before you try to solve the equation.
2	**Learn the quadratic formula** You will need to use the quadratic formula very often, in many different areas of mathematics, so make sure that you know it.
3	**When solving problems, make sure your answer makes sense** Always look at your answers to problems in the light of the original question. If there are two solutions, do they both make sense, or should one be discarded? Think carefully about the meaning of a negative solution, since this may or may not be a valid solution.

$$ax^2 + bx + c = 0$$

$$x_1 = \frac{-b + \sqrt{b^2 \quad 4ac}}{2a}$$

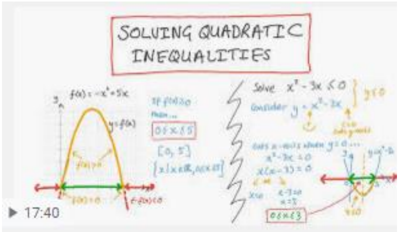
$$x_2 = \frac{-b + \sqrt{b^2 \quad 4ac}}{2a}$$

Inequalities

1	**Be careful when writing an inequality the other way round** Make sure that you reverse the inequality sign if you want to write the inequality the other way round ✗ **Wrong:** $3 < 2x+1$, $2x+1 < 3$ ✗ ✓ **Right:** $3 < 2x+1$, $2x+1 > 3$ ✓
2	**Be careful when multiplying an inequality** Make sure that you reverse the inequality sign if you multiply by a negative number. ✗ **Wrong:** $-x < 3-2x$, $x < -3+2x$ ✗ ✓ **Right:** $-x < 3-2x$, $x > -3+2x$ ✓
3	**Be careful when dividing an inequality** Make sure that you reverse the inequality sign if you divide by a negative number. ✗ **Wrong:** $-2x \geq 6x+4$, $x \geq -3-2$ ✗ ✓ **Right:** $-2x \geq 6x+4$, $x \leq -3-2$ ✓

Inequalities

4	**Sketching a graph or using a number line can help solve inequalities** When dealing with a quadratic inequality, always sketch a graph or a number line so that you can be sure that you are selecting the correct part as the solution.
5	**Make sure that the solution is the correct range of values!** With quadratic inequalities, make sure that you express the solution set correctly as either one range of values or two. If the solution is all values between –2 and 1: ✗ **Wrong:** $-2 < x$ or $x < 1$ ✗ ✓ **Right:** $-2 < x < 1$ ✓ If the solution is all values less than –2 or greater than 1: ✗ **Wrong:** $1 < x < -2$ ✗ ✓ **Right:** $x < -2$ or $x > 1$ ✓



Simultaneous Equations

1	**Be careful with signs when using the elimination method** It's very easy to make mistakes!
2	**Think about which method to use** If one equation gives, say, y in terms of x, it is usually easier to use the substitution method rather than the elimination method. When one equation is quadratic, you must always use substitution.
3	**Always check your solution** Just substitute your solution into both of the original equations to make sure that it fits
4	**Remember that for non-linear simultaneous equations there may be more than one solution** When you solve simultaneous equations where one is linear and one is quadratic, you should normally end up with two solutions unless: there is a repeated root (in which case the graph of the linear function is a tangent to the graph of the quadratic) **or** there are no solutions (in which case the graph of the linear function does not cross or touch the graph of the quadratic).

Co-ordinate Geometry

1	**Draw a diagram** In most questions involving coordinate geometry, it is helpful to draw a sketch diagram. It does not need to be accurate, but it will help to give you a rough idea of the answer you might expect.
2	**Ensure you can calculate the gradient of the line correctly.** The gradient of a line, m, is given by $m = \frac{\text{change in } y}{\text{change in } x}$ The gradient, m, of the line joining two points, (x_1,y_1) and (x_2,y_2) is given by $m = \frac{y_2 - y_1}{x_2 - x_1}$
3	**Make sure you can calculate the y-intercept of a straight-line graph.** The y-intercept of a line is where it crosses the y-axis. It is the value of y when x = 0.
4	**Make sure you understand how the standard straight-line equation works.** An equation which can be written in the form y=mx + c represents a straight line. m is the gradient and c is the y-intercept.

Co-ordinate Geometry

5	**Make sure you understand the conditions on the gradients of lines for the lines to be parallel or perpendicular.** If two lines have gradients m_1 and m_2 then: The lines are parallel if $m_1 = m_2$ The lines are perpendicular if $m_1m_2=-1$ (i.e. if $m_2 = -\frac{1}{m_1}$).
6	**Make sure you understand and can remember how to calculate the distance between two points** The distance, d, between two points, (x_1, y_1), and (x_2, y_2), is given by $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ This is just from applying Pythagoras's theorem
7	**Make sure you understand and can remember how to calculate the midpoint of the line between two points.** The coordinates of the midpoint, M, of the line joining (x_1, y_1), and (x_2, y_2), are given by $\text{M} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
8	**Make sure you can calculate the equation of a straight line.** - from the coordinates of two points on it. - from its gradient and the coordinates of a point on it.

Circles

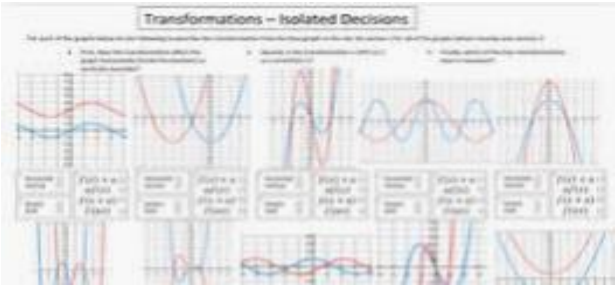
1	**Draw a diagram** In most questions involving coordinate geometry, it is helpful to draw a sketch diagram. It does not need to be accurate, but it will help to give you a rough idea of the answer you might expect.
2	**Make sure you know the standard circle equations** The general equation of a circle, centre (0,0) and radius r is: $x^2 + y^2 = r^2$ The general equation of a circle, centre (a, b) and radius r is: $(x-a)^2 + (y-b)^2 = r^2$
3	**Finding the intersection of a line and a curve** To find the coordinates of the point(s) where a line meets a curve, you solve the equations simultaneously. The condition for the line to be a tangent to the curve is that there is a repeated root. (For a line and a quadratic curve this means that the discriminant of the resulting quadratic equation is 0, i.e. b^2-4ac= 0). To find the coordinates of the point(s) where two curves meet you solve their equations simultaneously.
4	**You are expected to know these circle properties:** (i)the angle in a semicircle is a right angle (ii)the perpendicular from the centre of a circle to a chord bisects the chord (iii) the tangent to a circle at a point is perpendicular to the radius through that point These circle properties are often useful in examination questions. Keep them in mind when answering questions involving circles

Sketching Graphs of Functions

1	**Make sure that you know the basic rules about polynomial graphs** A polynomial of degree n crosses the x axis at most n times and has at most n–1 turning points. A repeated root means that the graph touches the x-axis at this point.
2	**Make sure that you know what a reciprocal graph looks like** You should recognise and be able to sketch graphs of the form y=k/x, where k is a constant. You should also know what is meant by an asymptote.
3	**Know what is meant by proportionality** If y is directly proportional to x, then y=kx, where k is a constant. If y is inversely proportional to x, then y=k/x.
4	**Make sure that you know what is expected in a sketch graph** If you are asked to sketch a graph, you do NOT need to plot points. You should show the basic shape of the graph, and label the points where the graph cuts the x-axis and the y-axis.
5	**Know how to find intersection points of graphs** Finding the intersection points of y = f(x) and y=g(x) is equivalent to solving the equation f (x) = g(x). Sketch graphs can be helpful to give you an idea of the number of roots and their approximate locations

Transforming Graphs

1	**Be careful with signs and directions when dealing with translations** - Remember that the transformation y = f (x) + a translates the graph of f (x) upwards if a is positive and downwards if a is negative. - Remember that the transformation f (x + a) translates the graph of f (x) to the left if a is positive and to the right if a is negative. Students often get this the wrong way round.
2	**Be careful with scale factors when dealing with stretches** - Remember that the transformation y = af (x) stretches the graph of f (x) by a scale factor a parallel to the y-axis. - Remember that the transformation y = f (ax) stretches the graph of f (x) by a scale factor 1/a parallel to the x-axis. So if a is greater than 1, the graph is compressed, and if a is less than 1, the graph is stretched. Again, students often get this the wrong way round.



Beckfoot

Maths	AS Pure	Year 12

enjoy learn succeed

Polynomials (e.g.Cubics) & Graphs

1	**When multiplying out brackets, make sure that you know how many terms there should be** You can find out how many by multiplying together the number of terms in each bracket, so that you know that you have not missed any.
2	**Make sure that you know the basic rules about polynomial graphs** A polynomial of degree n crosses the x axis at most n times and has at most n–1 turning points. A repeated root means that the graph touches the x-axis at this point.
3	**Don't lose easy marks when sketching polynomials** You will often be asked to sketch a polynomial which you have already factorised. Remember you are being asked for a sketch, so you should do this in your answer booklet and NOT on graph paper. You should certainly not be wasting time plotting points. Make sure that your graph does not stop at the axes, that you have shown all the points at which the graph cuts the axes (including the y-axis), and that your graph is the correct way up.
4	**Use a graphical calculator with caution** If you have a graphical calculator, you may find it useful to check the shape of a graph. However, you should understandthe principles of graph sketching and be able to show your reasoning. If you don't choose appropriate scales on a graphical calculator, you may not see important features. If you are asked to give exact values for intersections, reading them off a graph may not be adequate

Dividing & Factorising Polynomials

1	**Always check your work carefully** It is very easy to make silly mistakes when manipulating algebraic expressions.
2	**Make sure that you can divide polynomials with confidence** Try the different methods (long division, inspection or using a table) and choose the one that you feel most comfortable with. Get lots of practice until you feel really confident. Remember that you can check your answer by multiplying.
3	**Take care with signs** Be careful about signs when using the factor theorem: ✗ *Wrong* $f(2) = 0 \Rightarrow (x+2)$ is a factor ✗ ✓ *Right* $f(2) = 0 \Rightarrow (x-2)$ is a factor ✓
4	**When dividing a polynomial, remember that you can check by multiplying** You can also check the solutions of an equation by substitution.

Binomial Theorem

$$(a+b)^n = \binom{n}{0}a^n b^0 + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{n}a^0 b^n$$

$$\text{where } \binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

Binomial Expansion

1	**When using the binomial expansion, make sure that you remember to raise the whole term to the appropriate power** e.g. in the expansion of $(1 + 2x)^n$, remember that $(2x)^r = 2^r x^r$. ✗ *Wrong:* $(2+3x)^3 = 2^3 + 3\times 2^2 \times 3x + 3\times 2\times 3x^2 + 3x^3 = 8 + 36x + 18x^2 + 3x^3$ ✗ ✓ *Right:* $(2+3x)^3 = 2^3 + 3\times 2^2 \times (3x) + 3\times 2\times (3x)^2 + (3x)^3 = 8 + 36x + 54x^2 + 27x^3$ ✓
2	**Make sure that you can use the formula for binomial coefficients confidently** You need to know what is meant by ${}_nC_r$-this could be tested in your examination, and you may need to show that you know this formula rather than just using Pascal's triangle.
3	**Make sure you can find specific binomial coefficients efficiently** If asked to find a particular term in a binomial expansion, don't do the full expansion (which would waste a lot of time), just find the coefficient you need, making sure you use the right binomial coefficient. Also, remember that ${}_nC_r = {}_nC_{n-r}$

Trigonometric Functions

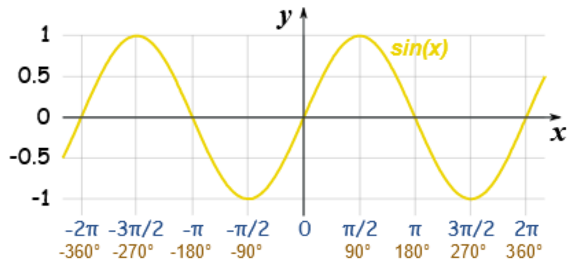
1	**You should know the exact values for certain angles** You will often need to use exact values of sin θ, cos θ and tan θ when θ= 0°, 30°, 45°, 60° or 90°, not the rounded values from your calculator.
2	**You must be able to accurately draw trigonometric graphs and know their properties** Make sure you can draw accurate sketches of the graphs of y= cos θ, y = sinθ and y = tanθ, and that you know their symmetries and periodic properties.

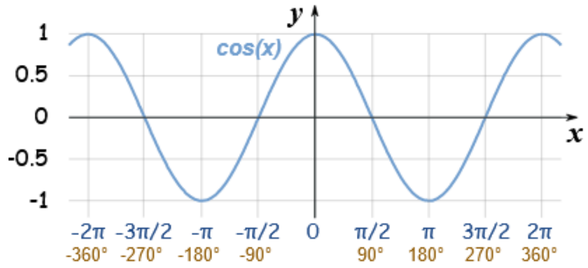
Trigonometric Equations

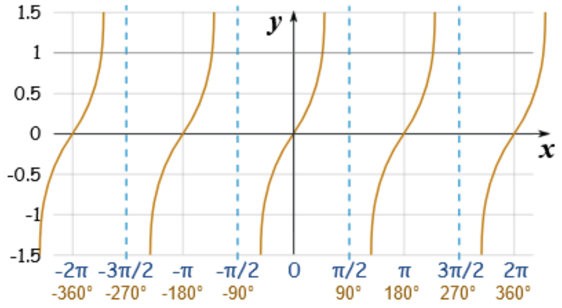
1	**Check that the roots are in the range asked for** Make sure that you check what range the roots should lie in.
2	**Remember to factorise instead of cancelling** Never cancel terms like sin θ or cos θ. Always factorise instead. For example, in an equation like sinθ – sinθcosθ=0, do not cancel out the term sinθ because you will lose the roots to the equation sinθ = 0. Instead, take out the factor sinθ to give sinθ (1 - cosθ) = 0.
3	**You must be able to draw trigonometric graphs accurately and know their properties** Make sure you know how to sketch the graphs of y= cos x, y= sin x and y= tanx and their properties
4	**Be careful when solving equations of the form sinax = k** Think about the number of roots you expect to obtain.

Sine and Cosine Rules

1	**Try geometry first when finding angles** Make sure that you check whether any missing angles can be found using geometry first.
2	**When finding angles, check for equivalent values** Make sure when you use the sine rule to find a missing angle, θ, that you check to see whether 180°–θ is also a solution.
3	**Check that all units are the same** Make sure you check that all the units are the same so you are not mixing, say, kilometres and metres.
4	**Check your calculator mode** Make sure your calculator is in degrees mode and not in radians mode.
5	**Be careful not to lose accuracy through rounding** Make sure you never round a number until you reach your final answer.







θ	0°	30°	45°	60°	90°
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\tan\theta$	0	$\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undefined

Maths	AS Pure	Year 12

Differentiation (Basics)

1 **Use notation carefully**
Make sure that you are familiar both with the notation dy/dx (used when you are given y as a function of x) and the notation f′ (x) (used when you are given a function f(x)).

2 **Use notation in the same way that it is used in the question**
Example: Differentiate v= t^2+ 2t.

✗ **Wrong** $\frac{dy}{dx} = 2t + 2.$

✓ **Right** $\frac{dv}{dt} = 2t + 2$, or

$$\frac{d}{dt}(t^2 + 2t) = 2t + 2.$$

The expression you are differentiating has variables v and t, not y and x, so you are finding $\frac{dv}{dt}$, not $\frac{dy}{dx}$.

3 **Draw a diagram if needed**
Questions on tangents and normals may go on to ask for other coordinate geometry work such as finding where lines cross. If this is the case, a diagram is very helpful

Differentiation (Basics)

4 **When calculating a gradient or tangent to a curve, make sure you get the coordinates the right way round**
Example: Find the gradient of the curve y= x^2 + 2x - 3 when it crosses the x axis.

✗ **Wrong** $\frac{dy}{dx} = 2x + 2$. When $x = 0$, $\frac{dy}{dx} = 2 \times 0 + 2 = 0$.

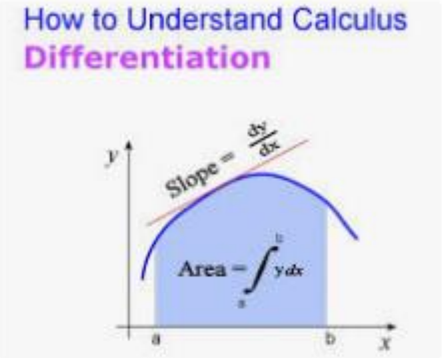
✓ **Right** $\frac{dy}{dx} = 2x + 2$. Curve crosses x axis when $y = 0$,

$\Rightarrow x^2 + 2x - 3 = (x - 1)(x + 3) = 0$

$\Rightarrow x = 1, \frac{dy}{dx} = 2 \times 1 + 2 = 4,$

or $x = -3, \frac{dy}{dx} = 2 \times (-3) + 2 = -4.$

5 **Remember the relationship between the gradients of perpendicular lines**
When finding the gradient of a normal, you need to first find the gradient of the tangent using differentiation, and then use the relationship m_1m_2 = -1. Always show clearly that you are using this relationship.



Negative and Fractional Powers

1 **Be careful when differentiating negative and fractional powers**
Make sure that you subtract 1 from the index correctly, especially when dealing with negative numbers.

2 **Make sure that you write products and quotients in an appropriate form when differentiating**
e.g. $(x-1)\sqrt{x}$ must be written as $x^{\frac{3}{2}} - x^{\frac{1}{2}}$

before differentiating.

$\frac{1-x}{x^2}$ must be re-written as $\frac{1}{x^2} - \frac{1}{x}$

Recall that $a^{-m} = \frac{1}{a^m}$

$$\left(\frac{a}{b}\right)^{-m} = \frac{a^{-m}}{b^{-m}} = \frac{\frac{1}{a^m}}{\frac{1}{b^m}} = \frac{1}{a^m} \times \frac{b^m}{1} = \frac{b^m}{a^m} = \left(\frac{b}{a}\right)^m$$

So that $\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$

Beckfoot

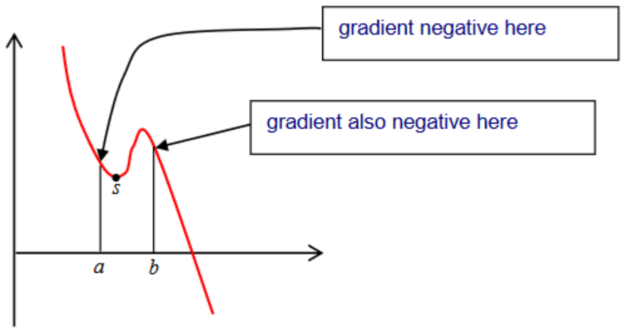
Maths	AS Pure	Year 12

enjoy learn succeed

Maximum and Minimum Points

1 **Take care which side of the stationary point you test the gradient**
When identifying whether a stationary point is a maximum or minimum by testing the sign of gradient either side of the stationary point, make sure you work from left to right, so you find the gradient at a value of x BEFORE the stationary point first, then at a value of x AFTER the stationary point.

2 **When testing the gradient either side of a stationary point, make sure the points you test are close enough to the stationary point you are investigating**
Otherwise, if there are two stationary points very close together, you may come to the wrong conclusion when identifying the stationary point.



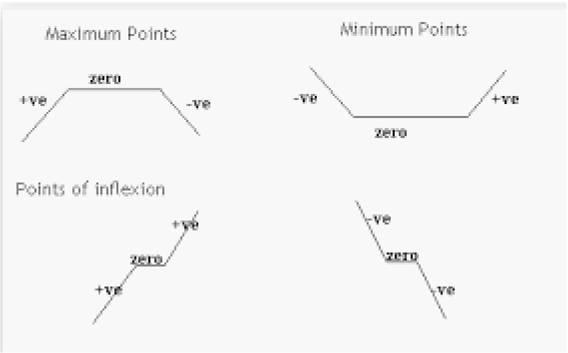
S is a minimum point between a and b, but the gradient is negative at a and negative at b. This error is caused by there being two stationary points close together, both of which are between a and b.

Maximum and Minimum Points

3 **Remember that you can't use the second derivative test when both the first and second derivatives are zero**
If the second derivative is zero, use the gradient method

4 **In a maximum or minimum problem, make sure that you answer the question**
For example, in a problem involving maximising the volume of a shape, make sure that you calculate the actual maximum volume obtained from the stationary value if you are asked for this.

5 **Draw diagrams**
When dealing with problems involving maximising or minimising, you should draw a clear diagram, which will help you to write down appropriate equations



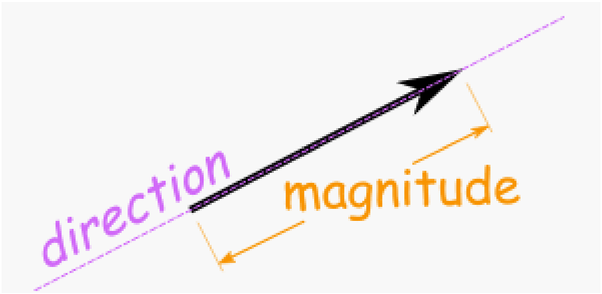
Working with Vectors

1 **Use vector notation correctly**
Remember that in handwriting you should underline vectors, or in the case of a vector joining two points, use an arrow above, e.g. $\overrightarrow{AB}$

2 **Make sure you know how to find the resultant of two vectors**
To find the resultant of two or more vectors simply add them together.

3 **Make sure you know how to find the vector joining two points**
The vector $\overrightarrow{AB}$ is found by $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

4 **Make sure that you know how to find a unit vector**
To find a unit vector in the same direction as a given vector, **a**, you divide by the magnitude, $|\mathbf{a}|$



Beckfoot

Maths	AS Pure	Year 12

enjoy learn succeed

Integration Basics

Don't muddle up the formulae for differentiation and integration

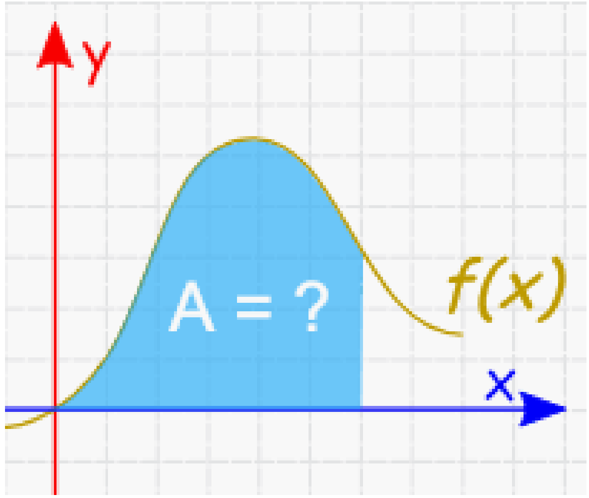
Example: Given that $\frac{dy}{dx} = x^3$, find y

✗ **Wrong** $\frac{dy}{dx} = x^3 \Rightarrow y = 3x^2$ ✗

✓ **Right** $\frac{dy}{dx} = x^3 \Rightarrow y = \frac{x^4}{4} + c$ ✓

Remember the arbitrary constant c

Always remember to put in the arbitrary constant –you will lose marks in an examination if you miss it out!



Finding the area under a curve

1 **Remember to integrate when calculating a definite integral!**

Don't forget to actually carry out the integration. The expression you write in the square brackets must be the integrated function.

Example: Evaluate: $\int_0^1 (x^3+1)\,dx$

✗ **Wrong** $\int_0^1 (x^3+1)\,dx = [x^3+1]_0^1 = (1^3+1)-(0+1) = 1$ ✗

✓ **Right** $\int_0^1 (x^3+1)\,dx = [\frac{1}{4}x^4+x]_0^1 = (\frac{1}{4}+1)-(0+0) = \frac{5}{4}$ ✓

2 **Don't add '+c' inside the square brackets**

Remember, you don't need an arbitrary constant when evaluating a definite integral.

3 **Be careful not to mix up indefinite and definite integrals**

Definite integrals have limits of integration – numbers on top and bottom of the integral sign, whereas indefinite integrals don't!

Example:

$\int x^2\,dx = \frac{1}{3}x^3 + c$ - Indefinite

$\int_0^1 x^2\,dx = \left[\frac{x^3}{3}\right]_0^1 = = \frac{1}{3} - 0 = \frac{1}{3}$ - Definite

4 **Be careful with signs**

When you are working out a definite integral it is very easy to make mistakes with signs, especially when dealing with negative limits. Use brackets to make your working clear.

5 **Be clear whether you are dealing with an area or just a definite integral**

If you are dealing with an area, this may be above or below the x-axis. For an area below the x-axis, the value of the definite integral will be negative. Since an area cannot be negative, you should give your final answer in such cases as positive. However, if you are simply asked to find a definite integral, with no reference to area, then you do not need to think about graphs at all, but just give the answer as calculated directly from the definite integral, which may be positive or negative

Fractional and Negative Indices

1 **Be careful when integrating negative and fractional powers**

Make sure that you add 1 to the index correctly, and remember that you must divide by the **new** index.

2 **Remember that you cannot yet integrate x^n for n= -1**

The rule for integrating x^n is valid for all values of n except for n = -1. You will learn to integrate 1/x in Year 13.

3 **Look out for discontinuities when finding the area under a graph**

Remember that you cannot integrate across a discontinuity.

Beckfoot

Maths	AS Pure	Year 12

enjoy learn succeed

Exponential Functions & Logs

1	**Make sure that you know the equivalent log relationships** It can be difficult to develop a "feel" for logarithms. Keep the equivalent relationships $\log_a b = c \Leftrightarrow a^c = b$ firmly in mind, remembering that the base of the logarithm is also the base of the index. The value of $\log_a b$ is the answer to the question: "What power must I raise a to in order to get b?"
2	**Remember the log laws** Make sure that you know the laws of logarithms. They are often useful for simplifying expressions.
3	**Remember that exponentials and logarithms are inverses of each other** This is important in solving equations. Equations involving exponentials can be solved by taking logs of both sides, and equations involving logs can be solved using powers

Natural Logs & Exponentials

1	**Learn and be confident using the laws of indices and logarithms** Make sure that you know the rules of logarithms and of indices, so that you can manipulate expressions involving exponentials and logarithms confidently.
2	**Make sure that you remember that the exponential and logarithm functions are the inverses of each other** Remember that the exponential function and the natural logarithm function are inverse functions; so you can "undo" an exponential function by using natural logarithms, and you can "undo" a natural logarithm by using exponentials.
3	**Be careful with signs** In an exponential decay model, be careful not to lose the minus sign. Remember that the logarithm of a number less than 1 is negative, and you can rewrite e.g. $\ln\frac{1}{2}$ as $-\ln 2$

Modelling Curves with Logs

1	**You should be able to convert polynomial functions into linear equations using logs** Make sure that you can show how the relationships $y = kx^n$ and $y = ka^x$ can be written in the form $y = mx + c$ by using logarithms.
2	**Make sure you plot the right quantities against each other** Check the linear form of the relationship to see what you need to plot $y = kx^n \Rightarrow \log y = \log k + n\log x$ Plot $\log y$ against $\log x$ $y = ka^x \Rightarrow \log y = \log k + x\log a$ Plot $\log y$ against x
3	**Remember to use inverse logs where appropriate to find the unknown constant** Make sure you know the relationships between the unknown constants and the gradient and intercept of the graph that you have plotted. $y = kx^n \Rightarrow \log y = \log k + n\log x$ Gradient = n, intercept = $\log k$ $y = ka^x \Rightarrow \log y = \log k + x\log a$ Gradient = $\log a$, intercept = $\log k$

Three main laws:

$$\log_a x + \log_a y = \log_a xy$$

$$\log_a x - \log_a y = \log_a \left(\frac{x}{y}\right)$$

$$\log_a (x^k) = k\log_a x$$

Special cases:

$$\log_a a = 1 \quad (a > 0, a \neq 1)$$

$$\log_a 1 = 0 \quad (a > 0, a \neq 1)$$

$$\log\left(\frac{1}{x}\right) = \log(x^{-1}) = -\log(x)$$