

Statistics Statistical Measures

1 Mean for grouped data
AKA 'estimated mean'.

Because data is grouped we find a mid-point which we then treat as our data.

length, L, cm	Frequency	Midpoint	<i>fx</i>
$0 < L \leq 10$	21	<i>5</i>	<i>105</i>
$10 < L \leq 20$	11	<i>15</i>	<i>165</i>
$20 < L \leq 30$	31	<i>25</i>	<i>775</i>
$30 < L \leq 40$	12	<i>35</i>	<i>420</i>
$40 < L \leq 50$	25	<i>45</i>	<i>1125</i>
<i>+ 100</i>			<i>+ 2590</i>

Calculate an estimate of the mean length of the fish.

$$2590 \div 100$$

$$25.9 \text{ cm}$$

2 Median for grouped data

Height (x cm)	Frequency
$0 < x \leq 10$	3
$10 < x \leq 20$	7
$20 < x \leq 30$	12
$30 < x \leq 40$	31
$40 < x \leq 50$	27

Frequency total $\div 2$ then count down the frequency total until we get to the number.

$$\frac{80}{2} = 40 \text{ value}$$

$$30 + \frac{10}{31} \times 10$$

$$35.806$$

3 IQR : UQ – LQ

(Interquartile range = upper quartile – lower quartile)

Upper Quartile (UQ) = 75%
Lower Quartile (LQ) = 25%

Eg. Find the IQR of 1, 2, 3, 4, 5, 6, 7

$$\text{IQR} = 6 - 2 = 4$$

Number Indices & Standard Form

1 Negative powers (Indices)

$$x^{-2} = \frac{1}{x^2}$$

$$x^{-3} = \frac{1}{x^3}$$

2 Fractional powers (Indices)

$$64^{\frac{2}{3}}$$

$$= \left((64)^{\frac{1}{3}} \right)^2$$

$$= \left(\sqrt[3]{64} \right)^2$$

$$= 4^2$$

$$16$$

3 Estimate powers & roots

Estimate:

$$\sqrt{28}$$

$$25 < 28 < 36$$



$$\sqrt{25} < \sqrt{28} < \sqrt{36}$$



$$5 < \sqrt{28} < 6 \checkmark$$

Number Indices & Standard Form

4 Ordinary form to standard form

5800000

$$= 5.8 \times 10^6$$

5 Multiplying and dividing standard form

$$(3 \times 10^5) \div (2 \times 10^3)$$

$$1.5 \times 10^2$$

$$(4.6 \times 10^8) \times (3.2 \times 10^3)$$

$$= (4.6 \times 3.2) \times (10^8 \times 10^3)$$

$$= 14.72 \times 10^{11}$$

Our answer isn't in standard form.

$$= 1.472 \times 10^{12}$$

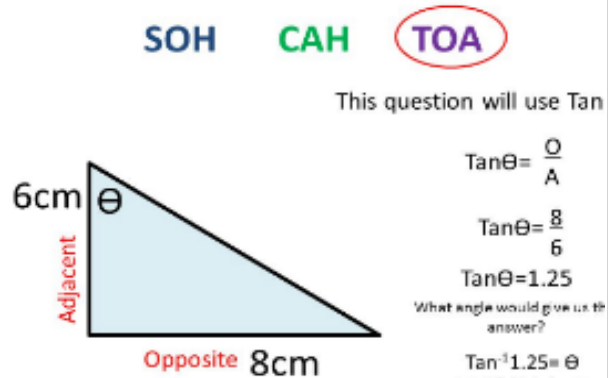
6 Product rule for counting

Eg. Katie has 52 different playing cards. She gives one card to Grace, one to Bill and one to Jenny. In how many ways can she do this?

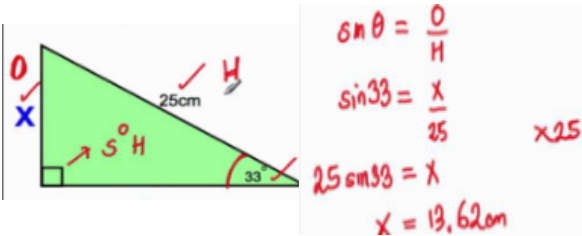
$$52 \times 51 \times 50 = 132600$$

Geometry & Measure – Trigonometry I

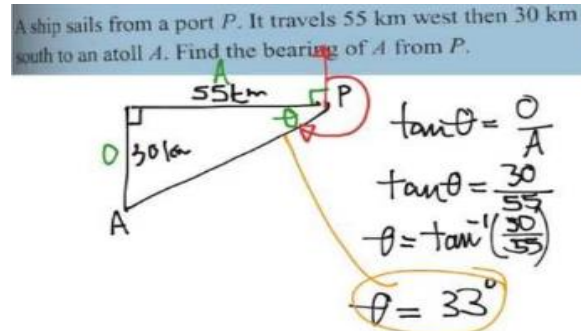
1 Find the missing angle



2 Find the missing side

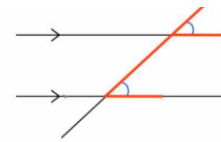


3 Trig with bearings



Geometry & Measure - Area & Angles

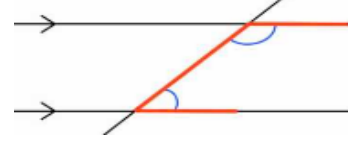
1 Corresponding angles are equal



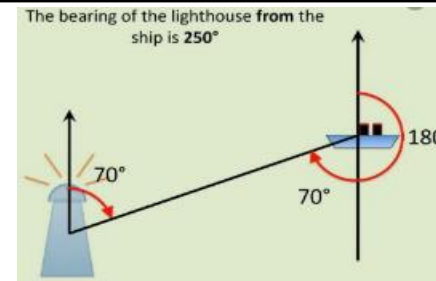
2 Alternate angles are equal



3 Interior angles add to 180°



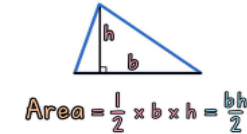
4 Bearings



4 Exact trig values

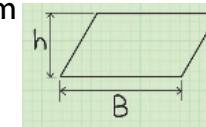
	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	—

5 Areas(& perimeter): Triangle

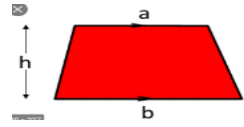


Parallelogram

$$A = Bh$$

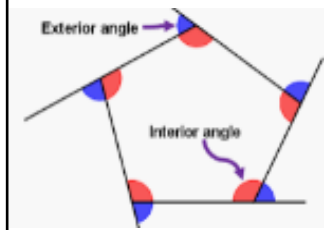


Trapezium

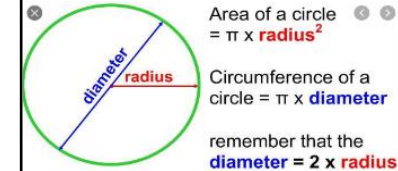


$$\text{Area} = \frac{1}{2} (a + b) \times h$$

6 Exterior / Interior angles of polygons: Sum of the interior angles = 180(n-2) Exterior angle of a regular polygon = 360/number of sides



7 Circle: area & circumference



Key Vocabulary

1	Mean, median, mode	Forms of averages – finding a middle value in a set of data
2	Trigonometry I	Finding missing angles/sides inside right angled triangles

Algebra Iteration / Linear Graphs

1 Growth/decay & compound interest

Emily invests £8000 in the bank for 4 years. It earns compound interest of 3% per year. $\times 1.03$

Calculate the total amount of money that Emily has in the bank after 4 years.

8000×1.03^4

8000×1.03^4

9004.07048

initial \times multiplier ^{time}

2 Approx. solution using iteration: using the previous answer to find the next answer.

Starting with $x_0 = 0$, use the iteration formula $x_{n+1} = \frac{1}{3} - \frac{x_n^2}{3}$ three times to find x_1, x_2 and x_3

$x_1 = \frac{1}{3} - \frac{0^2}{3} = \frac{1}{3}$

$x_1 = 0$
 $x_2 = 1/3$
 $x_3 = 8/27$

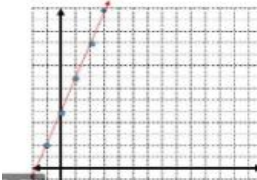
3 Linear graphs

Equation: $Y = 3x + 5$

Table:

X	-1	0	1	2
Y	2	5	8	11

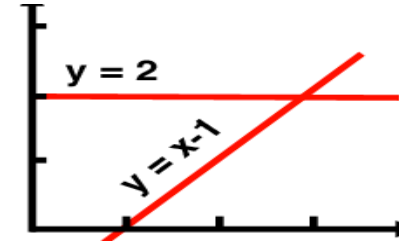
Graph:



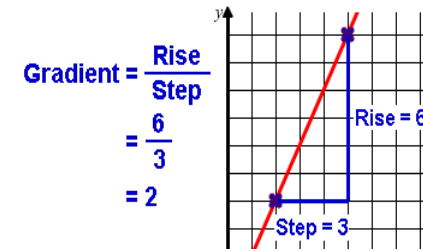
When the function is linear, then there is a common difference between each of the y values

4 Solve where 2 lines intersect

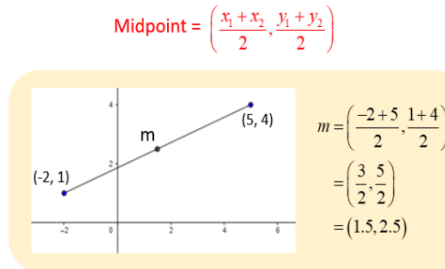
Solution (3,1)



5 Gradient of a straight line



6 Midpoint of a straight line



7 Linear equation from 2 points

$(x_1, y_1) \wedge (x_2, y_2)$

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - (-4)}{3 - 2} = \frac{10}{1}$

$y - y_1 = m(x - x_1)$

$y - (-4) = 10(x - 2)$

$y + 4 = 10x - 20$

$y = 10x - 24$

8 Equation of parallel line through a given point

Eg. A straight line has the equation $y = -2x - 3$. Find the equation of the parallel line passing through the point (1,3)

$y - y_1 = m(x - x_1)$

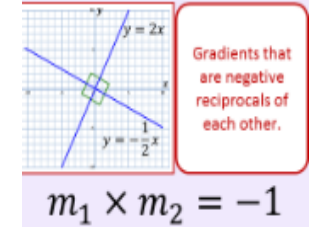
$y - 3 = -2(x - 1)$

$y - 3 = -2x + 2$

$+3 \quad +3$

$y = -2x + 5$

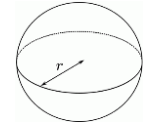
9 Use $y = mx + c$ to identify perpendicular lines



Key Vocabulary

1 Surface area sphere

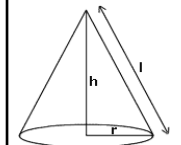
$s/a = 4\pi r^2$



2 Surface area cone

Full surface area $= \pi r l + \pi r^2$

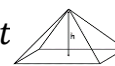
Curved surface area only $= \pi r l$





REMEMBER: sometimes you may need to calculate the slant of the cone (l) using Pythagoras.

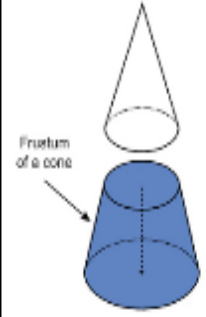
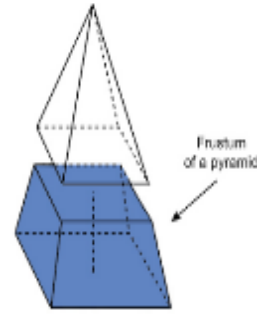
3 Volume Pyramid

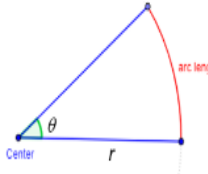
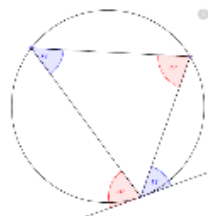
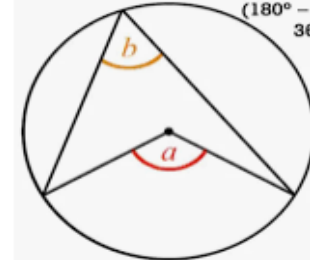
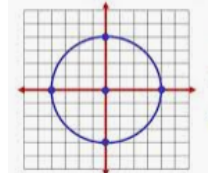
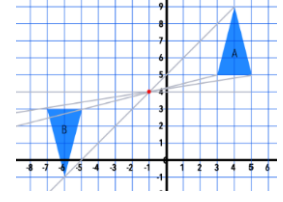
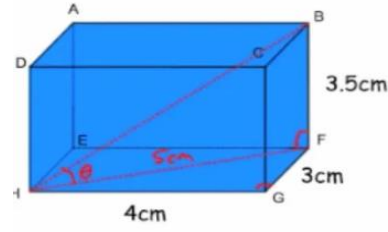
$\frac{1}{3} \times (\text{base area}) \times \text{height}$



Number Surds / Percentages	
1	Rationalise: x top & bottom by the surd $\frac{3}{\sqrt{5}} = \frac{3}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} \quad (\sqrt{5} \times \sqrt{5} = \sqrt{25} = 5)$ $= \frac{3\sqrt{5}}{5}$
2	Expand brackets $2(5 + \sqrt{3}) = 10 + 2\sqrt{3}$
3	Geometric sequences with surds $n=1 \rightarrow (\sqrt{2})^1 = \sqrt{2} \checkmark$ $n=2 \rightarrow (\sqrt{2})^2 = \sqrt{2} \times \sqrt{2} = 2 \checkmark$ $n=3 \rightarrow (\sqrt{2})^3 = \sqrt{2} \times \sqrt{2} \times \sqrt{2} = 2\sqrt{2} \checkmark$ $n=4 \rightarrow (\sqrt{2})^4 = \sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{2} = 4 \checkmark$ $n=5 \rightarrow (\sqrt{2})^5 = \sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{2} = 4\sqrt{2} \checkmark$ $\sqrt{2}, 2, 2\sqrt{2}, 4, \dots$
4	% increase / decrease Percent Increase = $\frac{[\text{new value}] - [\text{old value}]}{[\text{old value}]}$ Percent Decrease = $\frac{[\text{new value}] - [\text{old value}]}{[\text{old value}]}$
5	Multiplier in successive percentages Successive Percentage Changes An amount is increased by 20% then increased again by 30%. Find the overall percentage change. $\rightarrow \times 1.2 \rightarrow \times 1.3 \rightarrow$ $\rightarrow \times 1.56 \rightarrow$ Equivalent to a 56% increase overall.

Ratio & Proportion	
1	Set up & solve growth/decay problems <p>W is directly proportional to F and when $W = 24$, $F = 6$. Find the value of W when $F = 10$.</p> <div> $W = kF$ Substitute $W = 24$, $F = 6$ $24 = 6k$ $4 = k$ $W = 4F$ </div> <div> $W = 4F$ When $F = 10$, $W = 4 \times 10$ $W = 40$ </div>
2	Direct and Indirect Proportion graphs <div> Direct Proportion $y \propto x$ $y = kx$ for a constant k  </div> <div> Inverse Proportion $y \propto \frac{1}{x}$ $y = \frac{k}{x}$ for a constant k  </div>
3	Fraction - ratio In a class there are 5 boys and 8 girls. Express this as a ratio and fraction. <div> boys girls 5 : 8 $\frac{5}{13}$ are boys $\frac{8}{13}$ are girls </div>

Geometry Area & Volume	
1	Convert area units $\text{Km}^2 \xrightarrow{\times 1000^2} \text{m}^2 \xrightarrow{\times 100^2} \text{cm}^2 \xrightarrow{\times 10^2} \text{mm}^2$ $\div 1000^2 \quad \div 100^2 \quad \div 10^2$ $5\text{km}^2 = ? \text{m}^2$ Need to $\times 1000^2$ $5 \times 1000 \times 1000 = 5\,000\,000 \text{m}^2 \checkmark$
2	Convert volume units $\text{Km}^3 \xrightarrow{\times 1000^3} \text{m}^3 \xrightarrow{\times 100^3} \text{cm}^3 \xrightarrow{\times 10^3} \text{mm}^3$ $\div 1000^3 \quad \div 100^3 \quad \div 10^3$ $1200\text{cm}^2 = ? \text{m}^2$ Need to $\div 100^2$ $1200 \div 100 \div 100 = 0.12 \text{m}^2 \checkmark$ $3\text{m}^3 = ? \text{cm}^3$ Need to $\times 100^3$ $3 \times 100 \times 100 \times 100 = 3\,000\,000 \text{cm}^3 \checkmark$
3	Surface area of prisms Find the area of all the faces & add them up!
4	Frustum volume or surface area  

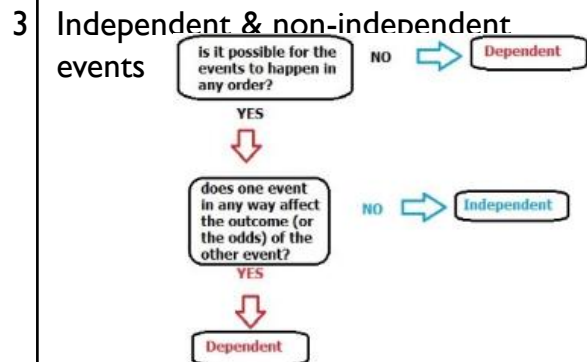
Geometry Circles Triangles Trig		
1	Arc length & sector area	 <div>$\text{arc length} = \frac{\theta}{360^\circ} \times 2\pi r$$\text{area of sector} = \frac{\theta}{360^\circ} \times \pi r^2$</div>
2	Alternate segment theorem	
3	Prove circle theorems	 <div>$(180^\circ - 2w) + (180^\circ - 2y) + a = 360^\circ$$360^\circ - 2w - 2y + a = 360^\circ$$a - 2w - 2y = 0$$a = 2w + 2y$$a = 2(w + y)$$w + y = b$$\text{so } a = 2b$</div>
4	Equation of a circle	 <div>$x^2 + y^2 = r^2$<p>r is the radius</p>$x^2 + y^2 = 16$<p>radius is 4 units</p></div>
5	Enlarge by a negative SF – This flips the shape. Eg SF -1 about (-1,4)	
6	Prove it's a right angle triangle	A triangle has the sides 3cm, 4cm and 6cm. Is this a right angle triangle? $a^2 + b^2 = c^2$ $3^2 + 4^2 = 6^2$ $9 + 16 = 36$ 25 \neq 36 (they are not equal) It is not a right angle triangle.
7	3D Trig	<p>3D Trigonometry</p> <p>Calculate the angle $\angle BHF$</p>  <div>$a^2 + b^2 = c^2$$3^2 + 4^2 = FH^2$$9 + 16 = FH^2$$25 = FH^2$$5 = FH$</div>
Key Vocabulary		
1	Experimental Vs. Theoretical probability	<div>Experimental probability is the result of an experiment or simulation after a large number of times.</div> <div>Theoretical probability is what is expected to happen based on the possible outcomes, assuming equally likely events.</div>
Number Fractions Decimals		
1	+ - x ÷	$= \frac{9 \times 1}{7 \times 5} = \frac{9}{35}$ $1\frac{2}{7} \div 5 = \frac{9}{7} \div \frac{5}{1} = \frac{9}{7} \times \frac{1}{5}$
2	Recurring decimal to fraction	$x = 0.5454545454 \dots$ $100x = 54.5454545454 \dots$ $99x = 54$ $99x = 54$ $x = \frac{54}{99} = \frac{6}{11}$
4	Reciprocal	$0.5 = \frac{1}{2}$ $\left(\frac{1}{2}\right) \frac{2}{1} = \underline{2}$ $\left(\frac{4}{3}\right) \frac{3}{4} = \underline{1}$
5	Upper lower bounds	<div>Area = $wh = ?$</div> <div>$w = 11.7\text{cm}$ (nearest mm)</div> <div>$h = 6.4\text{cm}$ (nearest mm)</div> <div>What are the upper and lower bounds of the area of the rectangle?</div> $11.65 \leq w < 11.75$ $6.35 \leq h < 6.45$ <div>Lb area = 11.65×6.35 Ub area = 11.75×6.45</div>

Probability

Colour	Probability
Blue	$\frac{1}{2}$
Yellow	$\frac{1}{4}$
Red	$\frac{1}{8}$
Green	$\frac{1}{8}$

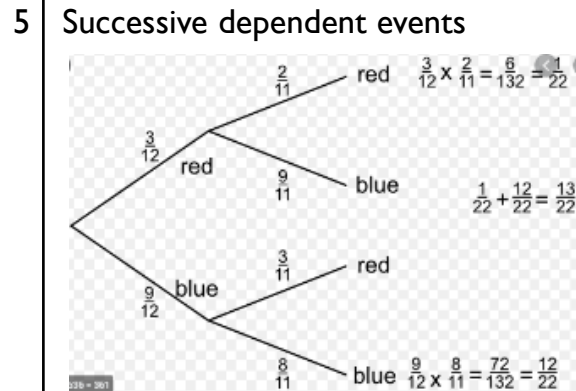
We calculate an estimate using:
probability \times **number of tries**
 In 20 spins, we would expect:
 Yellow $\rightarrow \frac{1}{4} \times 20 = 5$ **5 yellows**

Item	Frequency	Relative frequency
1	4	4/20 (or 20%)
2	5	5/20 (or 25%)
3	5	5/20 (or 25%)
4	2	2/20 (or 10%)
5	4	4/20 (or 20%)
Total	20	



4 Successive independent events

What is the probability of 2 heads on 2 successive throws

$$P(h) \times p(h) = 0.5 \times 0.5 = 0.25$$


5 And Or Rule

And: multiply \times
 Or: add $+$

6 Conditional probability

Probability which depends on a previous event. Eg if I choose a card but don't replace it and then choose another, the probability will change.

Algebra Inequalities & Equations

1. Solve inequalities

$$\begin{aligned} -3 &\leq 2x - 1 \leq 5 \\ +1 &\quad +1 \quad +1 \\ \hline -2 &\leq 2x \leq 6 \\ \frac{-2}{2} &\leq \frac{2x}{2} \leq \frac{6}{2} \\ -1 &\leq x \leq 3 \end{aligned}$$

2. Find all the integer solutions which satisfy this inequality:

$$\underline{-1 \leq x \leq 3}$$

-1, 0, 1, 2, 3

3. Solve with unknown both sides

$$\begin{aligned} 4m - 3 &< 2m + 6 \\ -2m &\quad -2m \\ \hline 2m - 3 &< +6 \\ +3 &\quad +3 \\ \hline 2m &< 9 \\ m &< \frac{9}{2} \end{aligned}$$

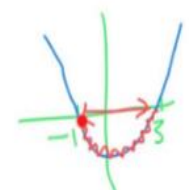
4. Quadratic inequalities & graph

Solve $x^2 - 2x - 3 < 0$

$$(x-3)(x+1) < 0$$

$x = 3$ $x = -1$

$-1 < x < 3$



5. Solve fractional equations

$$\begin{aligned} \text{Solve } \frac{5}{x-2} &= \frac{3}{x} \\ \frac{5}{x-2} &= \frac{3}{x} \\ 5x &= (x-2)(3) \\ 5x &= 3x - 6 \\ 2x &= -6 \\ x &= -3 \end{aligned}$$

6. Inverse functions

HINT :Change the subject of the formula

$$\begin{aligned} x &= (y-6)^2 + 4 \\ \sqrt{x+4} &= \sqrt{(y-6)^2 + 4} \\ \sqrt{x+4} &= y-6 \\ \sqrt{x+4} + 6 &= y \\ f^{-1}(x) &= \sqrt{x+4} + 6 \end{aligned}$$

7. Composite Functions

$$\begin{aligned} fg(x) &= 2(x-2) + 3 \\ &= 2x - 4 + 3 \\ &= \underline{2x - 1} \\ gf(x) &= (2x+3) - 2 \\ &= 2x + 3 - 2 \\ &= \underline{2x + 1} \end{aligned}$$

Algebra Simultaneous Equations, Quadratics Equations & Formulae

1 Solve simultaneous equations

Via elimination

Use the elimination method to solve the given simultaneous equations

$$\begin{array}{rcl} 5x + y & = & 20 \quad (*) \\ 4x + 5y & = & 37 \quad (**) \end{array}$$

Substitute $x = 3$ into $5x + y = 20$

$$5(3) + y = 20$$

$$15 + y = 20$$

$$y = 5$$

$\therefore x = 3, y = 5$

4 Factorise & solve

2 numbers that X to give 12 and + to give 7

$$x^2 + 7x + 12$$

$$(x + 3)(x + 4)$$

7 Solve via quadratic formula

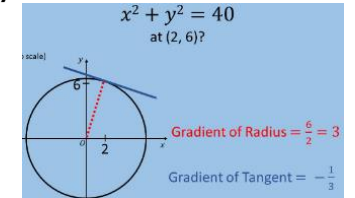
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Eg $2x^2 + 11x + 6 = 0 \Rightarrow a=2 \quad b=11 \quad c=6$

$$x = \frac{-11 \pm \sqrt{11^2 - 4 \times 2 \times 6}}{2 \times 2} \quad x = \frac{-11 \pm \sqrt{73}}{4}$$

$$x = -0.614 \text{ or } -4.886 \text{ (3dp)}$$

10. Tangent to circle equation

To find equation we use $y = mx + c$


$$M = -\frac{1}{3} \text{ \& sub (2,6)}$$

$$6 = -\frac{1}{3}(2) + c$$

$$6 + \frac{2}{3} = c \quad c = 6.67$$

$$y = \frac{1}{3}x + 6.67$$

2 Solve simultaneous equations

Via substitution

① $3x + 2y = 21$

② $y = x + 3$

A) Substitute y and solve to find x.

$$3x + 2(x + 3) = 21$$

$$3x + 2x + 6 = 21$$

$$5x + 6 = 21$$

$$5x = 15$$

$$x = 3$$

B) Input x to find y.

$$y = (3) + 3$$

$$y = 6$$

1) $y = x^2 - x - 6$

2) $y = 6 - 2x$

$$x^2 - x - 6 = 6 - 2x$$

$$x^2 + x - 12 = 0$$

$$(x - 3)(x + 4) = 0$$

$$x - 3 = 0 \text{ or } x + 4 = 0$$

$$x = 3 \text{ or } x = -4$$

Substitute both values of x into equation (1) or (2) and find both possible values of y.

$$y = 0 \text{ or } y = 14$$

$$(3, 0) \quad (-4, 14)$$

5 Factorise & solve a difference of 2 squares

$$a^2 - b^2 = (a + b)(a - b)$$

$$x^2 - 9 = (x + 3)(x - 3)$$

$$x = -3 \quad x = 3$$

8 Complete the square
Turning point (9, -1)

$$x^2 - 18x + 80 = 0$$

$$(x - 9)^2 - (-9)^2 + 80 = 0$$

$$(x - 9)^2 - 81 + 80 = 0$$

$$(x - 9)^2 - 1 = 0$$

$$(x - 9)^2 = 1$$

$$x - 9 = \pm \sqrt{1}$$

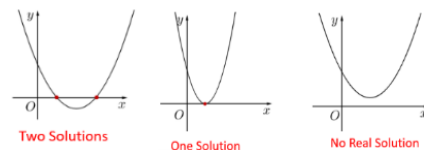
$$x = -1 + 9 \quad x = 8$$

$$x = +1 + 9 \quad x = 10$$

3 Solve quadratics via graphing

The x-intercepts of a graph are the solutions of the equation.

A quadratic equation can have one of three types of solutions:



6 Factorise & solve complex quadratics

 $3 \times 10 = 30$ Factors of 30 that + or - to make 11 are: $5 + 6 = 11$

$$3x^2 + 6x + 5x + 10$$

$$3x(x + 2) + 5(x + 2)$$

$$(3x + 5)(x + 2)$$

so $x = -5/3$ or $x = -2$

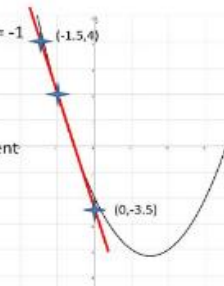
9 Gradient of tangent touching curve

Find the gradient of $y = x^2 - 3x - 2$ at the point $x = -1$

Draw a tangent at the point $x = -1$

Select 2 points on your tangent line

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3.5 - 4}{0 - -1.5} = \frac{-7.5}{1.5} = -5$$



11. Rearrange where a variable appears more than once

$$2(2p + m) = 3 - 5m$$

$$4p + 2m = 3 - 5m$$

$$4p + 2m + 5m = 3$$

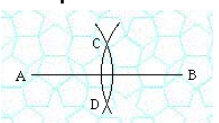
$$4p + 7m = 3$$

$$7m = 3 - 4p$$

$$m = \frac{3 - 4p}{7}$$

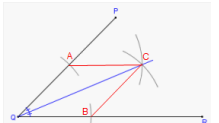
Geometry Construction

1 Perpendicular



1) Open the compass more than half of the distance between A and B, and scribe arcs of the same radius centered at A and B.
2) Call the two points where these two arcs meet C and D. Draw the line between C and D.
3) CD is the **perpendicular bisector** of the line segment AB.

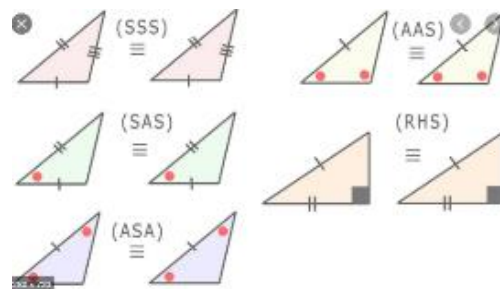
2 Angle Bisection



1) Place compass point on the vertex of the **angle** (point B).
2) Stretch the compass to any length that will stay ON the **angle**.
3) Swing an arc so the pencil crosses both sides (rays) of the given **angle**. ...
4) Place the compass point on one of these new intersection points on the sides of the **angle**.

3 Prove congruent Triangles

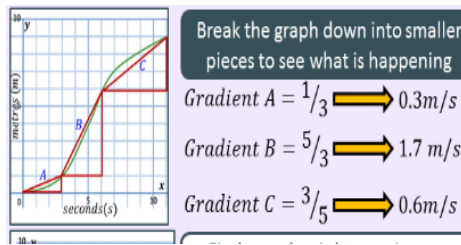
Know and understand the different tests for congruency.



(SSS) (SAS) (ASA) (AAS) (RHS)

Ratio Proportion Rates of Change Real Life Graphs


1 Calculate fastest average speed.



Break the graph down into smaller pieces to see what is happening

Gradient A = $\frac{1}{3} \Rightarrow 0.3\text{m/s}$
Gradient B = $\frac{5}{3} \Rightarrow 1.7\text{m/s}$
Gradient C = $\frac{3}{5} \Rightarrow 0.6\text{m/s}$

2 Velocity time graphs

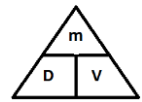


VELOCITY
TIME

$a+b+c+d+e = \text{Distance Travelled}$

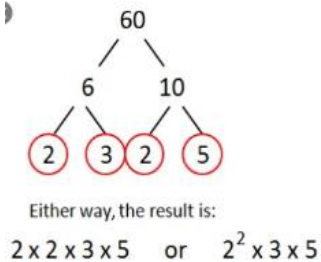
3 Density

density = $\frac{\text{mass}}{\text{volume}}$



Number

1. Prime factorisation

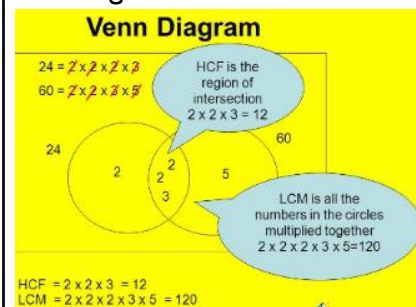


60
6 10
2 3 2 5

Either way, the result is:
 $2 \times 2 \times 3 \times 5$ or $2^2 \times 3 \times 5$

2. Using Venn to find HCF/LCM

Venn Diagram



$24 = 2 \times 2 \times 2 \times 3$
 $60 = 2 \times 2 \times 3 \times 5$

HCF is the region of intersection
 $2 \times 2 \times 3 = 12$

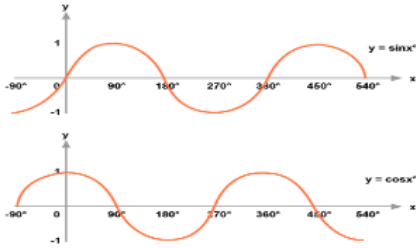
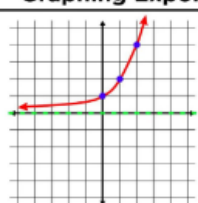
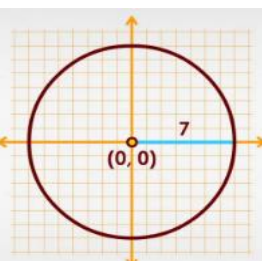
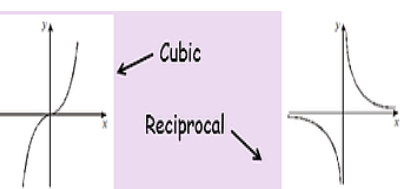
LCM is all the numbers in the circles multiplied together
 $2 \times 2 \times 2 \times 3 \times 5 = 120$

HCF = $2 \times 2 \times 3 = 12$
LCM = $2 \times 2 \times 2 \times 3 \times 5 = 120$

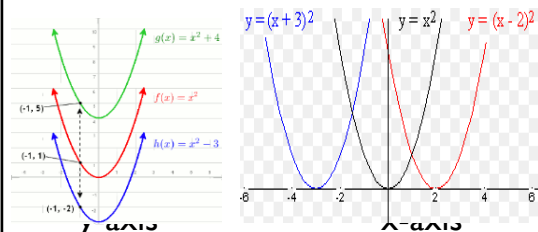
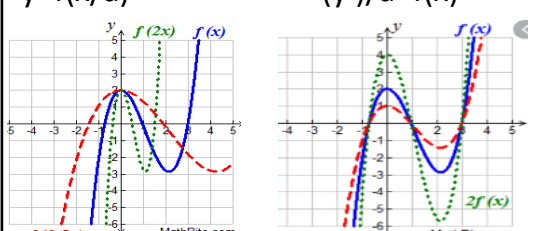
Key Vocabulary

1	Velocity	Is speed with direction.
2	Tangent	A straight line that touches a circle.
3	Roots or solutions	When we draw a quadratic equation, where the curve cuts through the x-axis are called the roots or solutions.
4	Gradient	Rate of change, so it could be the rate of water flow over time, or distance travelled over time.
5	Bisect	To mathematically accurately cut something in half e.g., an angle,
6	Prime factorisation	To break a number down into the primes we can multiply to make the original number.
7	Co-efficient	This is the number in front for example the co-efficient of this term $3x^2$ is 3.

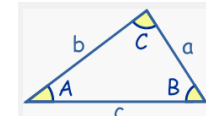
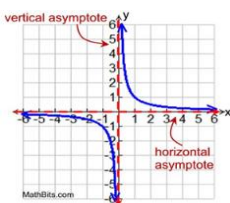
Algebra Cubic Circular, Exponential Functions

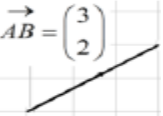
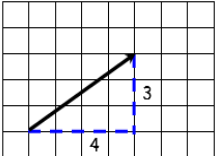
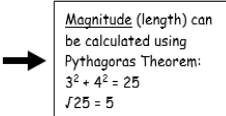
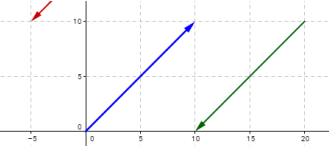
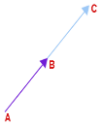
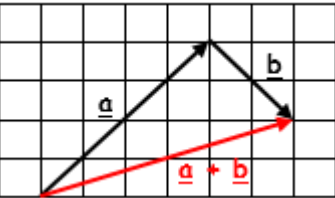
1	Sketch Sinx Cosx	
2	Sketch exponential graphs	Graphing Exponential Functions  <p>$y = 2^x$ horizontal asymptote: $y = 0$ the function is always positive <small>(raising to any exponent yields positive values)</small></p> <p> $2^{-1} = 1/2^1 = 1/2$ $2^{-2} = 1/2^2 = 1/4$ $2^{-3} = 1/2^3 = 1/8$ $2^{-4} = 1/2^4 = 1/16$ </p>
3	Graphs equations of circles	 <p> $x^2 + y^2 = r^2$ $x^2 + y^2 = 49$ </p>
4	Recognise cubic & reciprocal graphs	 <p> Cubic Reciprocal </p>

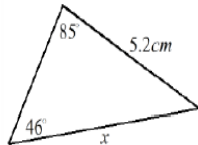
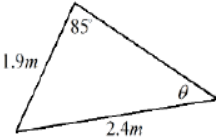
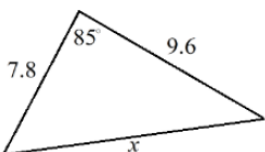
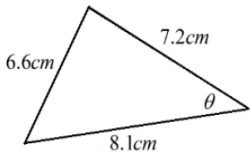
Transforming Functions & expanding brackets

5	Transforming graphs	
6	$y=f(x/a)$ $(y)/a=f(x)$	
	Single brackets	$5(x+3) + 6(x-4)$ $5x + 15 + 6x - 24$ $11x - 9$
	Double brackets	$(5x+2)^2$ $(5x+2)(5x+2)$ $25x^2 + 10x + 10x + 4$ $25x^2 + 20x + 4$
	Triple brackets	<p>Expand $(x+3)(x+5)(x+4)$</p> $(x^2 + 5x + 3x + 15)(x+4)$ $(x^2 + 8x + 15)(x+4)$

Key Vocabulary

1	Vectors	A quantity having direction as well as magnitude, in other words, size with direction!
2	Sine Rule For non-right angled triangles	Use with non right angle triangles . Use when the question involves 2 sides and 2 angles . For missing side: $\frac{a}{\sin A} = \frac{b}{\sin B}$ For missing angle: $\frac{\sin A}{a} = \frac{\sin B}{b}$ 
3	Cosine Rule For non-right angled triangles	Use with non right angle triangles . Use when the question involves 3 sides and 1 angle . For missing side: $a^2 = b^2 + c^2 - 2bccosA$ For missing angle: $cos A = \frac{b^2+c^2-a^2}{2bc}$
4	Exponential	Rapid change: exponential growth has growth with a power resulting in greater growth year on year, exponential decay is decay with a power resulting in greater decay year on year.
5	Asymptote	A curve/line never reaches zero (for $y = \frac{1}{x}$) instead it tends towards infinity. 

Geometry Vectors		
1	Column Vector	In a column vector, the top number moves left (-) or right (+) and the bottom number moves up (+) or down (-) Eg. $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ means '2 right, 3 up'
2	Vector 	A vector is a quantity represented by an arrow with both direction and magnitude . $\vec{AB} = -\vec{BA}$
3	Magnitude 	Magnitude is defined as the length of a vector. 
4	Parallel Vectors 	Parallel vectors are multiples of each other. Eg. $2\mathbf{a}+\mathbf{b}$ and $4\mathbf{a}+2\mathbf{b}$ are parallel as they are multiple of each other.
5	Collinear Vectors 	Collinear vectors are vectors that are on the same line . To show that two vectors are collinear , show that one vector is a multiple of the other (parallel) AND that both vectors share a point .
6	Resultant Vector 	The resultant vector is the vector that results from adding two or more vectors together. The resultant can also be shown by lining up the head of one vector with the tail of the other. if $\mathbf{a} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$ then $\mathbf{a} + \mathbf{b} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} + \begin{pmatrix} 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$

Geometry -Trigonometry 2		
1	Sine Rule – Missing Side 	$\frac{x}{\sin 85} = \frac{5.2}{\sin 46}$ $x = \frac{5.2 \times \sin 85}{\sin 46} = 3.75\text{cm}$
2	Sine Rule – Missing Angle 	$\frac{\sin \theta}{1.9} = \frac{\sin 85}{2.4}$ $\sin \theta = \frac{1.9 \times \sin 85}{2.4} = 0.789$
3	Cosine Rule – Missing Side 	$x^2 = 9.6^2 + 7.8^2 - (2 \times 9.6 \times 7.8 \times \cos 85)$ $x = 11.8$
4	Cosine Rule – Missing Angle 	$\cos \theta = \frac{7.2^2 + 8.1^2 - 6.6^2}{2 \times 7.2 \times 8.1}$ $\theta = 50.7^\circ$

Well done for getting this far & good luck with your GCSE!