Subject: Further Maths	Topic: Differentiation	Year Group: 11

An introduction to Differentiation

1

Remember to multiply out before differentiating a product Sums and differences of terms are easy to differentiate – just differentiate each part separately, and put the derivatives of the parts together. However, you cannot treat products in the same way. There is a product rule for differentiating, but this is not part of the syllabus. At this stage, you will need to multiply out the product first, then differentiate the result term-by-term.

Example: Differentiate $y = 3x^2(x^2 - 2)$.

✗ Wrong:
$y = 3x^2(x^2 - 2)$
$\frac{dy}{dx} = 6x \times 2x = 12x^2$ ✗

✓ Right:
$y = 3x^2(x^2 - 2)$
$y = 3x^4 - 6x^2$
$\frac{dy}{dx} = 12x^3 - 12x$ ✓

2

Rewrite quotients before differentiating As with products, quotients have a special rule for differentiating them. At this stage, you can only differentiate these by dividing them out into separate terms.

Example: Differentiate $y = \frac{x^3 + 2x^4}{4x^2}$.

✗ Wrong
$\frac{dy}{dx} = \frac{3x^2 + 8x^3}{8x}$ ✗

✓ Right
$\frac{x^3}{4x^2} + \frac{2x^4}{4x^2} = \frac{1}{4}x + \frac{1}{2}x^2$
$\frac{dy}{dx} = \frac{1}{4} + \frac{1}{2} \times 2x = \frac{1}{4} + x$

3

When calculating a gradient or tangent to a curve, make sure you get the coordinates the right way round
Example: Find the gradient of the curve y = x 2 + 2x – 3 where it crosses the x axis.

✗ Wrong
$\frac{dy}{dx} = 2x + 2$.
When $x = 0$, $\frac{dy}{dx} = (2 \times 0) + 2 = 2$. ✗

✓ Right
$\frac{dy}{dx} = 2x + 2$.
Curve crosses x axis when $y = 0$,
$\Rightarrow x^2 + 2x - 3 = 0$
$\Rightarrow (x-1)(x+3) = 0$
$\Rightarrow x = 1$ or $x = -3$
When $x = 1$, $\frac{dy}{dx} = (2 \times 1) + 2 = 4$
When $x = -3$, $\frac{dy}{dx} = (2 \times -3) + 2 = -4$ ✓

Further Differentiation

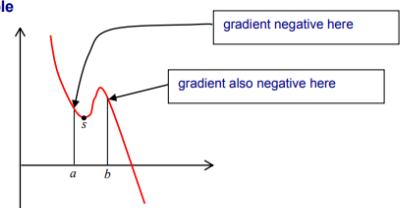
1

Take care which side of the stationary point you test the gradient When identifying whether a stationary point is a maximum or minimum by testing the sign of gradient either side of the stationary point, make sure you work from left to right, so you find the gradient at a value of x BEFORE the stationary point first, then at a value of x AFTER the stationary point. Remember: • For a maximum, the gradient is positive before the stationary point and negative after it • For a minimum the gradient is negative before the stationary point and positive after it.

2

When testing the gradient either side of a stationary point, make sure the points you test are close enough to the stationary point you are investigating Otherwise, if there are two stationary points very close together, you may come to the wrong conclusion when identifying the stationary point.

Example



s is a minimum point between a and b, but the gradient is negative at a and negative at b. The gradient test would therefore suggest that s is a point of inflexion. This error is caused by there being two stationary points close together, both of which are between a and b.

3

You can use the second derivative to identify maximum and minimum points. At maximum points the gradient is decreasing (going from positive to negative) as x increases and so the second derivative is negative. At minimum point the gradient is increasing (going from negative to postive) as x increases and so the second derivative is positive.

Subject: Further Maths	Topic: Matrices, Geometry and Functions	Year Group: 11

Matrix Arithmetic

1	**Check your answers carefully** It's easy to make careless mistakes in matrix multiplication.
2	**Make sure that you can do matrix multiplication confidently** This will also be needed in Section 2.
3	**Remember that matrix multiplication is not commutative** This means that AB ≠ BA. This is an easy mistake to make as we are all used to ordinary multiplication being commutative.
4	**Make sure you know the significance of the identity matrix** The identity r , $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ behaves in a similar way to the 1 in the multiplication of numbers: i.e. for any square matrix A, AI = IA = A.

Matrix Transformations

1	**Remember how to find the matrix representing a simple geometrical transformation** Remember the useful result that the image of the point (1, 0) gives the first column of the matrix, and the image of the point (0, 1) gives the second column of the matrix.
2	**Be careful with the order of transformations** Remember that in composite transformations, "A followed by B" is represented by the matrix BA.

Binomial Expansion

1	Make sure you know how to find the binomial coefficients. • When expanding (a + bx)n , don't forget to raise b to the same power as x. • If there is a negative involved, don't forget it!

Co-ordinate Geometry

1	(i) the angle in a semicircle is a right angle (ii) the perpendicular from the centre of a circle to a chord bisects the chord (iii) the tangent to a circle at a point is perpendicular to the radius through that point (iv) two tangents from a point to a circle are equal in length.

Geometrical Proof

1	Make sure you know all the theorems You need to be able to spot situations in which you can apply the theorems about angles in parallel and intersecting lines, and the circle theorems, so you need to know them thoroughly.
2	Mark angles on a diagram If you are asked to find an angle on a diagram, you may need to find other angles first. Write these on the diagram so that you can see clearly all the angles that you know
3	Explain your working In a proof question, you must explain what you are doing at each step. You need to say what theorem you are using, otherwise the examiner will not know whether you are using it correctly or just 'fudging'.

Functions

1	Make sure that you know what all of the terminology means Check that you know the meaning of all the terminology relating to functions such as range, domain, composed with and inverse function.
2	Remember how the notation for composition works. When the range of g is contained in the domain of f the function fog is defined as follows $f \circ g(x) = f(g(x))$. So $f \circ g(x)$ means g followed by f.

Subject: Further Maths	Topic: Extended Algebra	Year Group: 11

Limits

1	Make sure you know how to find the limit of a sequence

Example 6

The nth term of a sequence is given by $\frac{4n+1}{2n-3}$

Find the limit of the sequence.

Solution

As $n \to \infty$, $4n+1 \to 4n$.

$2n-3 \to 2n$.

$\frac{4n+1}{2n-3} \to \frac{4n}{2n} = 2$

So the limit of the sequence is 2.

$\to$ stands for 'tends to' which means 'becomes closer and closer to'. The symbol ∞ represents infinity. So 'as $n \to \infty$' means 'as n tends to infinity'.

Factor Theorem

1	**Find a method for factorising that suits you** Once you have found a factor of a cubic expression using the factor theorem, there are a number of different ways of dividing so that you can complete the factorisation. Several different methods are shown in the interactive resources on the website. Try some different methods and then stick with the one that you are most comfortable with.
2	**Take care with signs** Be careful about signs when using the factor theorem: ✗ Wrong f(2) = 0 ⇒ (x + 2) is a factor ✗ ✓ Right f(2) = 0 ⇒ (x – 2) is a factor ✓

Factorising, algebraic fractions and formulae

1	**Make sure that you are not making basic errors** Errors in algebra are very common. Sometimes these are just careless mistakes, but sometimes you may make errors because you have not understood a technique correctly. If you have problems with any technique in this section of work, read the worked examples very carefully and make sure that you understand each step. If you are not sure, make sure that you consult a teacher.
2	**Know what is meant by taking out a factor** When factorising, make sure that you understand that "taking out a factor" means dividing each term by that factor, NOT subtracting.
3	**Make sure that you are confident with algebraic fractions** If you are having trouble with algebraic fractions, you may find it helps to practise some numerical fractions first, so that you can be sure that you remember the techniques involved.
4	**Be careful with cancelling fractions** Remember that when you cancel fractions, you are dividing the numerator and denominator by the same thing. You have to divide each term by the same thing, and it may help to factorise numerator and denominator if possible so that you can see any factors. A fraction like 2 p p q + cannot be simplified, as p is not a factor of the whole denominator.
5	**Make sure that you can rearrange a formula with confidence** If you are stuck on rearranging a formula, try writing in numbers instead of all the letters except for the new subject, so that you have an equation with one unknown. Then think what you would do to solve the equation, and do the same thing to the formula that you are rearranging.
6	**Make sure the new subject only appears once** When changing the subject of a formula, make sure the new subject does not appear on both sides of the equals sign in the rearranged formula.

e.g. Make x the subject of $b = \frac{ax-2}{x}$

✗ WRONG

$b = \frac{ax-2}{x}$

$\Rightarrow bx = ax - 2$

$\Rightarrow x = \frac{ax-2}{b}$ ✗

x is still present on the RHS of the equation

✓ RIGHT

$b = \frac{ax-2}{x}$

$\Rightarrow bx = ax - 2$

$\Rightarrow bx + 2 = ax$

$\Rightarrow 2 = ax - bx$

$\Rightarrow 2 = x(a-b)$

$\Rightarrow \frac{2}{a-b} = x$

$\Rightarrow x = \frac{2}{a-b}$ ✓

Get the terms involving x on the same side of the equation

Isolate x by factorising

Pythagorean Triples

1	Know the triples such as 3,4,5 5, 12, 13 7, 24, 25