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Complex Numbers

1	**Make sure you get the statement of de Moivre's theorem right.** De Moivre's theorem says that $(cos\theta + isin\theta)^n = cosn\theta + isinn\theta$ for all integers n. It ***does not*** say, for example, that $cos^n\theta + isin^n\theta = cosn\theta + isinn\theta$ for all integers n. This is just one of numerous possible silly errors.
2	**Make sure that you don't get the modulus of an nth root of a complex number wrong.** Remember that $\|z^n\| = \|z\|^n$, and this applies not just to integer values of n, but includes rational values of n, as when taking roots of z.
3	**Make sure that you get the right number of nth roots of a complex number.** There should be exactly n of them. Remember two complex numbers which have the same moduli and arguments which differ by a multiple of 2π are actually the same number.
4	**Make sure you can work comfortably with both the exponential and modulus-argument forms.** Sometimes it is better to use the former and sometimes it's better to use the latter, you need to be comfortable with both.

Series

1	**Make sure you have the correct left-over terms when using the method of Differences.** After most of the terms cancel out, the left over terms may not necessarily be the first and the last, and there may be more than two. You need to write out the first few terms and the last few terms in full to spot the pattern.
2	**Check your answer by substituting n.** It is a good idea to substitute n = 1, and perhaps n = 2 as well, to check your result.
3	**Be careful when substituting into standard Maclaurin series.** Remember that if you are finding, for example, e^{2x} by substituting 2x into the standard series, that you must find $(2x)^2$, $(2x)^3$ etc: remember to find the power of 2 as well as the power of x!
4	**Make sure it's appropriate to use standard series.** For example, you can't easily use standard series to find the expansion for ln(cos x), because you would need to substitute the series for cos x into every term of the series for ln x to get all the constant terms, and so on. In cases like these, you need to use repeated differentiation and substitute into the general Maclaurin formula.

Sketching rational functions

1	**Factorise where possible.** Make sure that you always factorise both the numerator and the denominator if possible, and if they are not already given in factorised form. If you don't, you may miss vertical asymptotes or points where the graph cuts the x axis.
2	**When sketching related graphs, think about important points.** Think about points where the original graph crosses the x-axis, or has an asymptote, or there is a turning point, and decide what happens to the new graph at this point. Also think about the behaviour of the new graph as $x \to \pm\infty$.
3	**Don't rely on a graphical calculator.** Graphical calculators often do not give you a clear idea of the shape of this type of graph. Some of the features may be off the screen, and turning points near a horizontal asymptote may not be clear. Changing the axes to see a larger part of the graph may mean that features are not clear. However, your calculator may be useful for checking. If you know the features you are looking for, then you can confirm them using the calculator. For example, you could zoom in on points on the x-axis to check the behaviour of the tangent.

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Hyperbolic Functions

1	**Remember that using the definitions is often useful.** When faced with an equation like $coshx + 2sinhx = 1$, you might be tempted to think that you need to solve it in the same way as the equivalent trigonometric equation. However, writing the equation in terms of exponentials is often a useful method. Sometimes, however, this doesn't work out easily, and you may find that using an identity is more successful.
2	**Make sure that you know all the relevant techniques of differentiation and integration.** The calculus techniques you use in this unit are not new: you have used them all before in the context of different functions. If you are having any difficulty, look back to the calculus work A level Mathematics.
3	**Be careful when evaluating integrals and inverse hyperbolic functions.** You sometimes need to deal with some quite complicated logarithmic functions when using the inverse hyperbolic functions. It is very easy to make careless mistakes, so always check your work. If your calculator has the hyperbolic functions on it, you can use it to double-check your answer.

Improper integrals

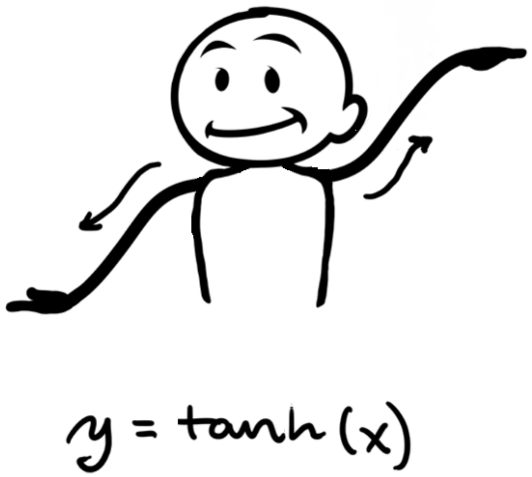
1	**Make sure you know what is meant by an improper integral.** You need to be able to recognise two types of improper integral: ones where one of the limits is infinity, and ones where the integrand is not defined at one of the limits or at some point in between the limits.
2	**Remember that improper integrals may or may not have values.** Some improper integrals can be evaluated; others are undefined.

Inverse trig functions

1	**Make sure that you apply the standard integrals correctly.** Remember in particular that the coefficient of x^2 must be 1, and that if it isn't you must take out a factor or use a substitution before carrying out the integration.
2	**Make sure that you are confident in completing the square so that the standard integrals can be applied.** Get plenty of practice in this if you are not confident with it!

Further Integration

1	**Look out for integrals which need partial fractions.** Make sure you recognise integrals which can be put into partial fractions, and make sure you know how to integrate each separate type of fraction.
2	**Look out for integrals where a trigonometric substitution may be needed.** For integrals which involve a power or root of $a^2 - x^2$, the substitution $x = asin\theta$ may be useful, and for integrals which involve a power or root of $a^2 + x^2$, the substitution $x = atan\theta$ may be useful.



Miss Mclean's Graph Dance

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Polar Coordinates

1	**Make sure you know how to find the area of a sector by integration.** Remember that the area of a sector is found by integrating $\frac{1}{2}r^2$ with respect to θ.
2	**Be careful with integration, in particular when integrating $\cos^2\theta$ or $\sin^2\theta$.** Remember the use of the double angle identity to integrate $\cos^2\theta$ or $\sin^2\theta$.
3	**Make sure that you use the correct limits of integration.** Always sketch the curve and use it to check the limits of the integration.

1st Order Differential Equations

1	**Be careful with the sign when modelling a situation using a differential Equation.** Always consider carefully whether the rate of change is positive or negative
2	**Be careful with notation.** Rate of change is denoted by $\frac{d}{dt}$. Always use the same letters for variables that are given in the question (for example don't change $\frac{dx}{dt}$ to $\frac{dy}{dx}$). If you change the letter used, then it may be difficult for the examiner to follow what you are doing. You can use the 'dot' notation if you like (i.e. $\dot{x}$ denotes $\frac{dx}{dt}$, $\ddot{x}$ denotes $\frac{d^2x}{dt^2}$), but do make sure that your dots are clear!
3	**Make sure that you include the arbitrary constant when integrating.** Remember that you only need an arbitrary constant on one side of the equation.
4	**Remember to include the modulus sign, if necessary, when integrating to give a logarithmic function.** Make sure that you think about whether it can be dropped, according to the context of the question.
5	**Always write the differential equation in the standard form before finding the integrating factor.** The standard form is $\frac{dy}{dx} + Px = Q$.
6	**Be careful with exponentials and logarithms when finding the integrating factor.** Remember $e^{\ln x} = x$ and $e^{a\ln x} = e^{\ln x^a} = x^z$
7	**Remember to multiply the right-hand side of the equation by the integrating factor.** This is an easy mistake to make!

Integrating Factor Method

- $\frac{dy}{dx} + P(x)y = Q(x)$
- $I.F = e^{\int P(x)dx}$

2nd Order Homogeneous Differential Equations

1	**Make sure you write down the auxiliary equation correctly.** Be especially careful if the differential equation is not written in the usual form: for example if the terms are not in the usual order.
2	**Remember that if the auxiliary equation has a root, λ, which is zero, this will result in a constant term in the solution.** A useful check is that the number of terms in the general solution should be the same as the order of the differential equation.
3	**Be sure to use the correct form of the general solution.** Make sure that you know each of the four cases shown in the table below for second order differential equations.

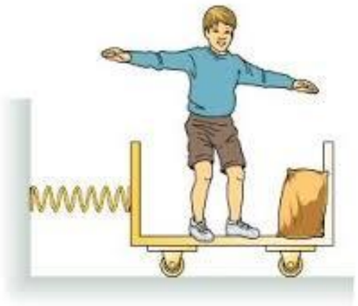
Roots of auxiliary equation	Form of general solution
Two distinct real roots α and β	$y = Ae^{\alpha x} + Be^{\beta x}$
One repeated root m	$y = (A + Bx)e^{mx}$
Pure imaginary roots $\pm ni$	$y = Asinnx + Bcosnx$
Complex roots $p \pm qi$	$y = e^{px}(Asinqx + Bcosqx)$

Non-Homogeneous 2nd order Differential Equations

1	**Make sure you use the correct form of trial function.** Be especially careful in the special case where the function on the right-hand side is of the same form as one of the complementary functions: remember to multiply the trial function by the independent variable (usually x or t) in these special cases.
2	**When using the initial conditions to find the values of the unknown constants, make sure you use the whole solution.** When you are dealing with a non homogeneous equation, you must find the full general solution first, consisting of the complementary function and the particular integral, and then substitute initial conditions into this full general solution to find the particular solution. Do not try to find the unknown constants when you have found the complementary function only!
3	**When using a substitution, remember to substitute back at the end.** If the original equation is given involving y and x, your final solution must be in terms of y and x only, and not involve any new variable such as u. Always substitute back to obtain the original variables.

Modelling Oscillations

1	**Make sure you define variables clearly.** When setting up an SHM equation from a situation such as a mass on an elastic string or spring, it is important to define your variables precisely and ensure that the position of the origin and the positive direction are clear. A good clear diagram is essential.
2	**The standard equation $\ddot{x} = -\omega^2 x$ is obtained only if the equilibrium position is used for $x = 0$.** Otherwise you need to write your equation in the form $\ddot{x} = -\omega^2(x - x_0)$. Then you can see that the substitution $y = x - x_0$ leads to the standard equation for y with centre of oscillation $y = 0$ i.e. $x = x_0$.
3	**Make sure you know what is meant by damping.** You should be able to determine whether a system exhibits light damping, heavy damping or critical damping.



Coupled Equations

1	**Use the correct variables.** Remember that when you differentiate the original equations, you are differentiating with respect to t, so differentiating y gives $\frac{dy}{dt}$
2	**Use the solution for one variable to find the solution for the other variable.** For example, if you have found a solution for x, find $\frac{dx}{dt}$ and substitute into the original equation for $\frac{dx}{dt}$ (if you try to use the other equation you will end up with more work!). If you are working with general solutions, then this means that the solution for y will be expressed in terms of the unknown constants that you used for x, so that you have just two unknown constants altogether. If you start again by eliminating x from the original equations, not only will you give yourself a lot more work, but you will also introduce two new unknown constants.
3	**Be careful when finding the values of the unknown constants.** Check your solution with the initial conditions.

Numerical Integration

1	**Make sure you use the formula for the mid-ordinate rule correctly.** Remember that you need to find the values of the function at the mid-points of the strips, not at the ordinates themselves.
2	**Make sure you use the formula for Simpson's rule correctly.** Don't mix up the numbers that need to be multiplied by 4 with those that need to be multiplied by 2. Also, remember to divide the whole answer by 3 (not 2, as for the trapezium rule)
3	**Remember that Simpson's rule can only be used for an even number of strips.** The trapezium rule can be used with any number of strips, but this is not the case for Simpson's rule.

Euler's Method

1	**Make sure you understand the notation.** You need to be familiar with the notation used in given formulae for finding the numerical solutions of first order differential equations of the form $\frac{dy}{dx} = f(x, y)$.
2	**Show your working.** Show your working In the exam you must show sufficient working so that the correct method is clear and method marks are not lost due to careless slips.
3	**Avoid rounding errors.** You will sometimes need to use a previously calculated value in a further calculation. Make sure that you write down enough decimal places in any intermediate solutions so that rounding errors do not affect your final answer. It is useful if you store intermediate answers in the memory of your calculator.

For $\frac{dy}{dx} = f(x)$ and small h, recurrence relations are:

Euler's method: $y_{n+1} = y_n + hf(x_n)$, $x_{n+1} = x_n + h$

For $\frac{dy}{dx} = f(x, y)$:

Euler's method: $y_{r+1} = y_r + hf(x_r, y_r)$, $x_{r+1} = x_r + h$

Improved Euler method: $y_{r+1} = y_{r-1} + 2hf(x_r, y_r)$, $x_{r+1} = x_r + h$

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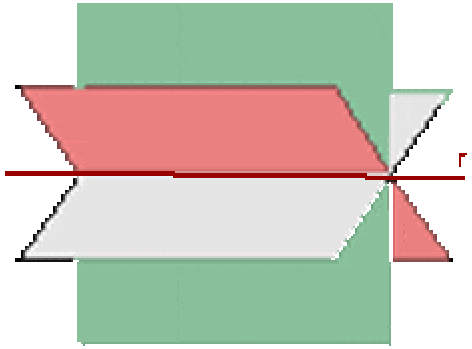
Inverse of a 3x3 Matrix

1	**Remember the place signs when finding cofactors.** Note that the place signs do not tell you the signs of the cofactors. They tell you whether or not you need to change the sign of the minor to obtain the cofactor.
2	**Use the matrix facility on your calculator to check.** If your calculator handles matrices, you should be able to use it to find a determinant or an inverse of a 3x3 matrix.
3	**Check inverses by multiplying the matrix by its inverse.** Remember that multiplying a matrix by its inverse gives the identity matrix. If it doesn't, you have made a mistake!

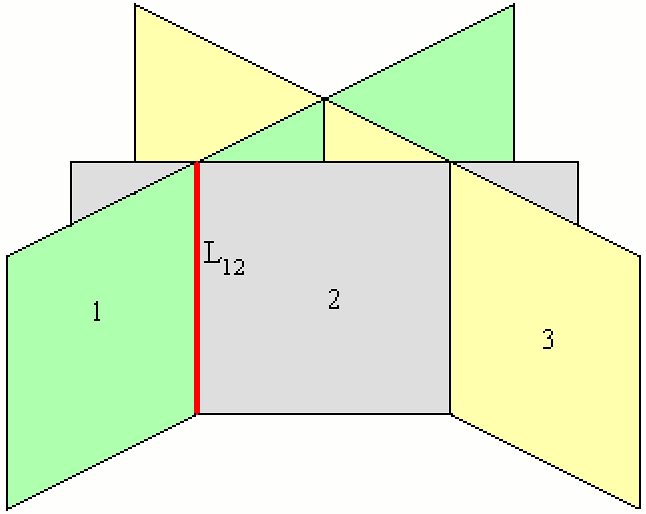
Matrices and Simultaneous Equations

1	**Be careful when solving matrix equations for which the matrix has no inverse.** It is tempting to immediately conclude that there is no solution, but in fact what this means is that there is no unique solution, and there may be either no solution or infinitely many solutions.
2	**Make sure you understand the connection between three planes and the solution of three simultaneous equations.** If three planes intersect at a point, there is a unique solution If the planes form a sheaf, they intersect along a line so there are infinitely many solutions If the planes form a triangular prism, or if two or more of them are parallel, there are no solutions



Factorising Determinants

1	**Remember the effects of operations you can use.** You can add or subtract any multiple of one row or column to another without changing the determinant. Multiplying any row or column by a constant results in the determinant also being multiplied by the same constant
2	**Look out for factors that can be taken out.** When working with algebraic matrices, the factors may not be immediately obvious.



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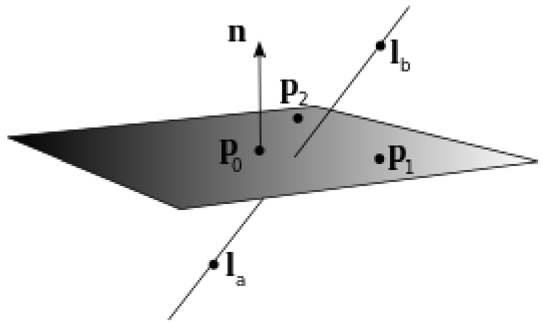
The Equation of a Plane

1	**Make sure you know the form of the equation of a plane.** The equation of the plane is $n_1x + n_2y + n_3z + d = 0$ where $d = -\boldsymbol{a}.\boldsymbol{n}$. The coefficients n_1, n_2 and n_3 give the the direction vector normal to the plane and **a** is the position vector of a point on the plane
2	**Make sure you know the different forms of the equation of a plane.** In scalar product form: **r.n = a.n** In Cartesian form: $n_1x + n_2y + n_3z = d$ where **d = a.n** . In both these cases **n** is a vector normal to the plane and **a** is the position vector of a point on the plane. In vector form: $r = \boldsymbol{a} + \lambda\boldsymbol{b} + \mu\boldsymbol{c}$ where **a** is the position vector of a point on the plane and **b** and **c** are both vectors which lie in the plane.
3	**Remember how to recognise the normal vector to a plane from its equation.** For a plane with Cartesian equation ax + by + cz = d, the normal vector is $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$. This is fundamental to all of your vector work. Make sure you know and understand it.

Lines and Planes

1	**Remember how to find the intersection of a line and a plane.** You need to use a general point on the line (i.e. coordinates of a point in terms of the parameter λ), and substitute this into the equation of the plane to find the value of λ.
2	**Remember how to find the angle between a line and a plane.** You can do this by finding the angle between the direction vector of the line and the normal to the plane. Remember that you must then subtract this from 90° to find the angle between the line and the plane.
3	**Remember that shortest distances always involve perpendicular lines.** The shortest distance from a point to plane is the length of a line segment perpendicular to the plane.



The Vector Product

1	**Use the vector product to find a vector perpendicular to two other vectors.** When you find the vector product of two vectors **a** and **b**, the result is a vector that is perpendicular to both **a** and **b**. This has some useful applications such as finding the equation of a plane.
2	**Remember that the vector product is anti-commutative.** Make sure that when manipulating expressions involving vector products, that you keep the vectors in the same order, or if you need to change the order, change the sign.
3	**Make sure you know the different forms of the equation of a line.** In vector form: **r** = **a** + λ**b** Vector product form: $(\boldsymbol{r} - \boldsymbol{a}) \times \boldsymbol{b} = 0$ Cartesian form: $\frac{x - a_1}{b_1} = \frac{y - a_2}{b_2} = \frac{z - a_3}{b_3}$ where **a** is the position vector of a point on the line, and **b** is a vector in the direction of the line.