Beckfoot

Further Maths	AS Pure	Year 12

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Matrices and Transformations	
1	**Check answers carefully** It's easy to make careless mistakes in matrix arithmetic.
2	**Make sure that you can do matrix multiplication confidently** This is fundamental to all work on matrices.
3	**Remember that matrix multiplication is not commutative** In general, AB ≠ BA. This is an easy mistake to make as we are all used to ordinary multiplication being commutative.
4	**Make sure that you are familiar with the matrices for simple transformations** You need to know the matrices for reflection in the x axis, the y axis and the lines y = x and y = -x, and the matrices for rotation through 90° or 180° about the origin. All these look a bit similar, with 0s, 1s and –1s, so make sure that you can work out what they are with a quick diagram if you're not sure.
5	**Make sure that you are also familiar with the matrices for enlargement and two-way stretches** These are quite easy to remember, with the numbers on the leading diagonal giving you the scale factors, and zeros in the other two positions.
6	**Make sure that you understand how a shear is defined** A shear has a fixed line (in the cases you will meet the fixed line will be either the x-axis or the y-axis). The shear can be defined by giving the fixed line and the image of a point not on the line. The shear factor is the distance moved by a point divided by its perpendicular distance from the fixed line.

7	**Make sure that you know the general rotation matrix** The matrix for a rotation of anticlockwise about the origin is, $\begin{pmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{pmatrix}$ and that you ...en if cos and/or sin are negati... can find the angle of rotation from the matrix, including cases where the angle is not in the first quadrant.
8	**Remember the useful result about the columns of a matrix** The image of the point I (1, 0) gives the first column of the matrix, and the image of the point J (0, 1) gives the second column of the matrix.
9	**Make sure you multiply matrices in the correct order for composite transformations** Remember that "transformation A followed by transformation B" is represented by the matrix BA.

Invariance, Determinants and Inverses	
1	**Remember that the origin is always an invariant point for a linear transformation** Either the origin is the only invariant point, or there are an infinite number of invariant points which all lie on the same straight line – a line of invariant points.
2	**Make sure that you know the difference between a line of invariant points and an invariant line** An invariant point is a point which is mapped to itself, so a line of invariant points is a line of points each of which is mapped to itself. An invariant line is a line of points each of which is mapped to a point which is also on the line (not necessarily itself). A line of invariant points is, of course, also an invariant line.
3	**Remember the rule for the inverse of a matrix product** For square matrices M and $(MN)^{-1} = N^{-1}M^{-1}$.
4	**Make sure you understand the significance of a zero determinant for a matrix transformation** For a matrix with zero determinant, all points on the plane are mapped to a straight line through the origin, and each set of object points which are mapped to a single image point all lie on a straight line.

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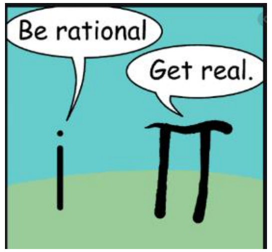
Complex Numbers

1	**Always simplify i^2** Remember that when you are working with complex numbers, you should always simplify i^2 to -1.
2	**Remember that zz* is always real** In particular, remember that you can use this in dividing complex numbers.
3	**Make sure that you know the condition for equality** For two complex numbers to be equal, the real parts must be equal and the imaginary parts must be equal.
4	**Make sure that you can plot complex numbers correctly on the Argand diagram** Remember in particular that the points z and z* are reflections of each other in the x axis, and that the points z and –z are rotations of each other through 180° about the origin.
5	**Make sure that you know how to show addition and subtraction in the Argand diagram** You need to understand that a complex number can be represented not only by a point in the Argand diagram, but alternatively by a vector.

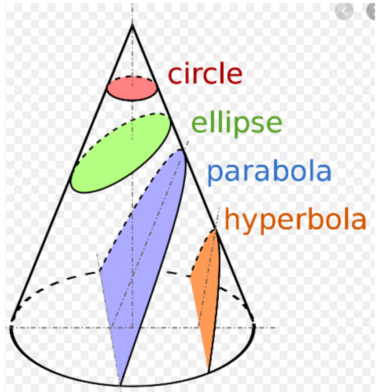


Roots of Polynomials

1	**Check your algebra** The most common problem in this topic is mistakes with algebra. The algebra can be quite complicated, but if you can do this then it will help you in all areas of mathematics.
2	**Make sure that you have learnt the relationships between roots and coefficients** In particular, make sure that you remember the pattern of alternating signs in the relationships between roots and coefficients.
3	**Be careful with the Σ notation** In particular, make sure that you know how many terms are involved in each case: for example, for a cubic then $\Sigma\alpha\beta$ has three terms, but for a quartic $\Sigma\alpha\beta$ has six terms.
4	**Remember that complex roots of polynomial equations with real coefficients always occur in conjugate pairs** This means that if you know one complex root, then you know another one. Note: this does not apply if the coefficients of the equation are not real!
5	**Make sure that you can divide a polynomial by a linear or quadratic factor** This is covered in AS Mathematics – look back at this work if you need to.
6	**Check your work carefully** It is easy to make mistakes in the algebra when solving polynomial equations.

Conics

1	**Learn the equations of the conics** You need to know the equations of the standard parabola, ellipse, hyperbola and rectangular hyperbola. Also make sure you know where they cross the axes, and in the case of the hyperbola, the equations of the asymptotes.
2	**Make sure you know how to deal with transformations** You need to be able to work with translations, stretches and reflections in the lines y x □ and y x
3	**Make sure that you know how to find out whether a line crosses a curve** Substituting the equation of a straight line into the equation of a conic gives you a quadratic equation to solve. If this quadratic equation has two roots, then the line crosses the conic once. If it has a repeated root, the line touches the conic (i.e. it is a tangent). If it has no real roots, then the line does not meet the conic.



Hyperbolic Functions

1	**Don't get mixed up between the properties of the hyperbolic functions and the circular functions** In particular, don't $\cos^2 x + \sin^2 x \equiv 1$ and $\cosh^2 x - \sinh^2 x \equiv 1$
2	**Make sure you know the graphs of the hyperbolic and inverse hyperbolic functions** Make sure that you can remember the graphs for sinh x, cosh x and tanh x. You can use these to find the graphs of the inverse hyperbolic functions.
3	**Remember that the graph of cosh x is symmetrical about the y-axis** This means that there are two values of x for every possible value of cosh x. Remember that using the logarithmic formula for the inverse cosh function will give you just the positive value for x.
4	**Be careful when evaluating inverse hyperbolic functions** You sometimes need to deal with some quite complicated logarithmic functions when using the inverse hyperbolic functions. It is very easy to make careless mistakes, so always check your work. If your calculator has the hyperbolic functions on it, you can use it to double-check your answer.

Summing Series

1	**Make sure you have the correct left-over terms when using the method of differences** After most of the terms cancel out, the left over terms may not necessarily be the first and the last, and there may be more than two. You need to write out the first few terms and the last few terms in full to spot the pattern.
2	**Check your answer by substituting for n** Whether using the method of differences or standard results to find a sum of the first n terms of a series, it is a good idea to substitute n = 1, and perhaps n = 2 as well, to check your result.
3	**Factorise where possible** When using standard results, there can be quite a lot of algebra involved in simplifying the result. Make sure you take out any common factors first, as this makes the algebra a lot simpler.

Maclaurin Series

1	**Be careful when substituting into standard Maclaurin series** Remember that if you are finding, for example, 2 e x by substituting 2x into the standard series, that you must find $(2x)^2$, $(2x)^3$ etc: remember to find the power of 2 as well as the power of x!
2	**Remember that some of the standard series are valid only for certain values of x** The ranges of validity are given in your formula book.

Proof by Induction

1	**Understand the concept** Make sure that you really understand the principle behind proof by induction. The Notes and Examples should help.
2	**Always think about what you are aiming for!** When you take the assumed result for n = k and add on the (k + 1)th term, you want to rearrange this to get the formula for n = k + 1. It may help to actually write down the result you are looking for.
3	**Be careful with algebraic manipulation It is easy to make mistakes.** Thinking about the result you are aiming for (see above) often helps, as it may give you a clue about what factors you could take out.
4	**Make sure that you write out the proof correctly** Remember that there are three steps involved, and you will lose marks if you don't, for example, write down the conclusion of the argument (Step 3).

Volume of Revolution

1 **Don't forget the π in the volume of revolution formula** Remember the formulae:

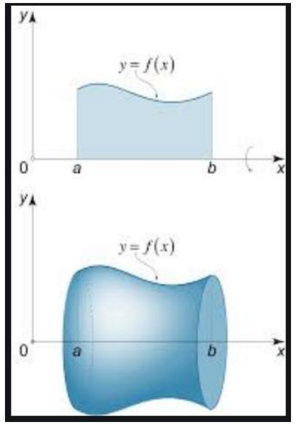
$$V=\int_a^b \pi y^2\,dx \text{ and } V=\int_c^d \pi x^2\,dy$$

2 **Make sure that you use the correct limits of integration** Remember that if you are rotating about the x-axis, the limits of integration must be x-coordinates, and if you are rotating about the yaxis, the limits of integration must be y-coordinates.

3 **Remember to integrate with respect to the correct variable** You need to substitute for x^2 or y^2 to do this. Example Find the volume of revolution of $y = x^2$ about the x-axis between x = 0 and x = 1

✗ ***Wrong*** $V=\int_0^1 \pi y^2\,dx = \pi\left[\frac{1}{3}y^3\right]_0^1 = \frac{1}{3}\pi$

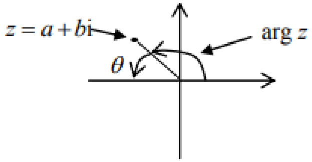
✓ ***Right*** $V=\int_0^1 \pi y^2\,dx = \int_0^1 \pi x^4\,dx = \pi\left[\frac{1}{5}x^5\right]_0^1 = \frac{1}{5}\pi$



Further Complex Numbers

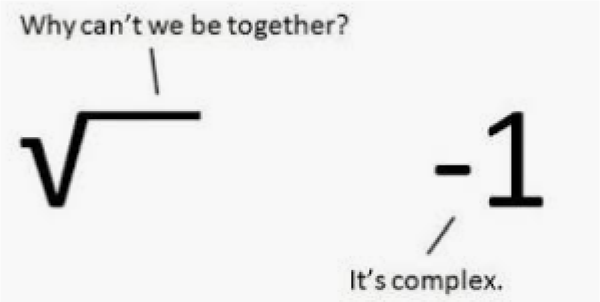
1 **Be very careful when you find the argument of a complex number** Always decide first which quadrant the complex number is in, and remember that when you have worked out the value of on your calculator, $\arctan\frac{y}{x}$

this will only be correct for complex numbers in the first and fourth quadrant. For the second quadrant, you need to add π, and for the third quadrant you need to subtract π. It's a good idea to make a rough sketch of the number on an Argand diagram, so you can 'see' the argument.



$\arg z = \pi - \theta$,
where $\theta = \arctan\left(\frac{b}{a}\right)$

2 **Use the modulus-argument form correctly** Remember that the modulus-argument form of a complex number must be of the form $r(\cos\theta + i\sin\theta)$ with r positive.



Loci in the Complex Plane

1 **You should recognise equations and inequalities which represent circles** Any set of points of the form |z – (a + bi)| = r is represented by a circle, centre a + bi, radius r.

2 **You should recognise equations and inequalities which represent perpendicular bisectors** Any set of points of the form

$$|z-(a+bi)| = |z-(c+di)|$$

is represented by the perpendicular bisector of the line joining the points a + bi and c + di. Don't mix this up with the circle locus!

3 **Make sure you show sets of points involving the argument correctly** Remember that for the set of points

$$\arg(z-(a+bi)) = \theta$$

the set of points is a half-line starting from the point z = a + bi. However the point z = a + bi is not included and should be shown by an open circle.

4 **Use the correct range for the argument** Remember that the possible values of arg z are given by $-\pi < \arg z \le \pi$

Make sure when drawing sets of points of the form

$\arg(z-(a+bi)) \le \theta$ or $\arg(z-(a+bi)) \ge \theta$

that you use the correct range for the argument.

5 **Be careful with inequalities** A set of points defined using an inequality represents a region. Remember that if < or > are used, the boundary of the region (a circle or a line) is not included and should be shown as a dotted line, but if ≤ or ≥ are used, the boundary is included and should be shown as a solid line.

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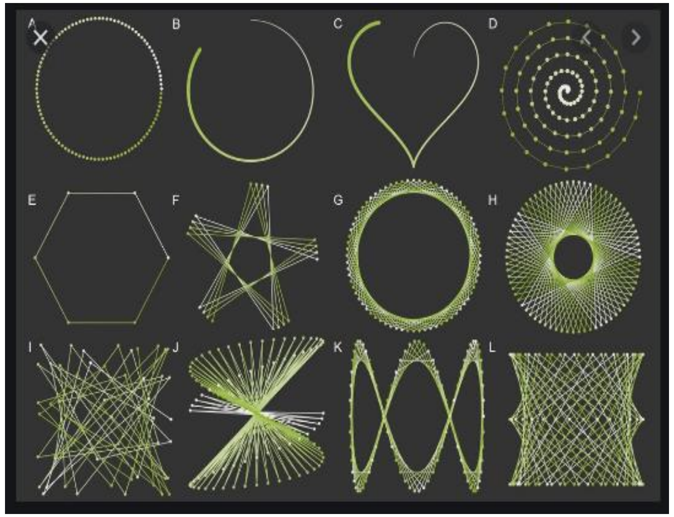
Polar Coordinates and Curves

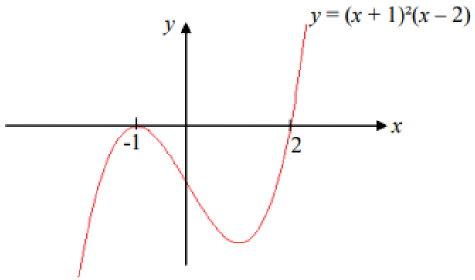
1	**Make sure you plot polar coordinates correctly** Always make sure that you check whether Ө is positive or negative.
2	**Make sure you change Cartesian coordinates into polar coordinates correctly** When finding Ө, make sure that you know which quadrant the point is in.
3	**Make sure that you sketch polar graphs correctly** Look for any points at which r is zero or takes its maximum or minimum value. Decide whether r is increasing or decreasing between these points.



Mr Kendall’s calculators can draw each and every one of these!

Rational Functions

1	**Factorise where possible** Make sure that you always factorise both the numerator and the denominator if possible, and if they are not already given in factorised form. If you don’t, you may miss vertical asymptotes or points where the graph cuts the x axis.
2	**Put all the information that you have on to your initial sketch** Remember when completing the sketch that the graph cannot cross the x axis at any point other than the points which you found in Step 1.
3	**Don’t be too hasty in completing the sketch** Make sure that there is only one possible way in which you can do it. If there isn’t, then obtain the extra information you need, such as the sign of y near the asymptotes.
4	**Make sure that you know how to find a turning point without using calculus** Remember that at a local maximum or minimum point, the graph touches a horizontal line y = k, so the equation f(x) = k (rearranged to form a quadratic in x) has a repeated root.



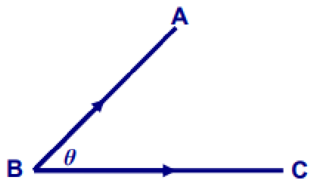
Inequalities

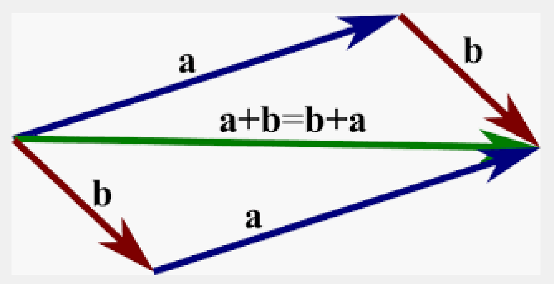
1	**Be careful to use the correct inequality sign in your answers** If the question involves < or >, the solution set should involve < or >. However, be careful when the question involves ≤ or ≥, as sometimes the answer may involve < or > (see point 2 below).
2	**Watch out for points where the function is undefined** Points which are undefined, such as at an asymptote, must not be included in your solution set.
3	**Remember that you cannot multiply through by a quantity which may be positive or negative** You can’t multiply through by an expression such as (x – 3), which is positive for some values of x and negative for others. However, you could multiply through by a quantity such as $(x – 3)^2$ which is always positive – but if you do, remember that in this case x = 3 must be excluded from the solution set.
4	**Watch out for points where a function becomes zero but does not change sign** If you want to solve an inequality of the form f(x) ≥ 0, an isolated point where the graph of y = f(x) touches the x axis must be included. For example, to solve the inequality $(x + 1)^2(x – 2) \geq 0$, the graph below shows that x ≥ 2 is part of the solution, but x = -1 is also part of the solution, since y = 0 when x = -1.

Scalar Product

1 **Remember that the scalar product of perpendicular vectors is zero** To show that two vectors are perpendicular just show that the scalar (or dot) product of the vectors is 0.

2 **Draw diagrams to make sure that you are using the right vectors** If you want to find angle ABC , the diagram below shows that you need to work out the angle between the vectors BA and BC





The Vector Equation of a Line

1 **Make sure you understand the relationship between vector and cartesian equations of lines:**

The line $\mathbf{r}=\begin{pmatrix}2\\-1\end{pmatrix}+\lambda\begin{pmatrix}3\\1\end{pmatrix}$ has a gradient of $\frac{1}{3}$.

The line $\mathbf{r}=\begin{pmatrix}3\\2\end{pmatrix}+\lambda\begin{pmatrix}1\\-2\end{pmatrix}$ has a gradient of -2.

2 **Make sure you know how to find the angle between two lines** To find the angle between two lines simply find the angle between the two direction vectors.

3 **Remember to watch your signs when converting between the vector and cartesian equations of a line.**

Example: The cartesian equation of the line $\mathbf{r}=\begin{pmatrix}-2\\1\\3\end{pmatrix}+\lambda\begin{pmatrix}3\\-5\\1\end{pmatrix}$

is $\frac{x-(-2)}{3}=\frac{y-1}{-5}=\frac{z-3}{1}$

which tidies up to give: $\frac{x+2}{3}=\frac{1-y}{5}=z-3$

4 **Be careful when writing down the Cartesian equation of a line which has one or two zeros in the direction vector.**

For example, you might think that the line $\mathbf{r}=\begin{pmatrix}2\\6\\-2\end{pmatrix}+\lambda\begin{pmatrix}0\\4\\-3\end{pmatrix}$ in Cartesian form is $\frac{x-2}{0}=\frac{y-6}{4}=\frac{z+2}{-3}$. However, division by 0 is undefined. For this line, the x-coordinate of all points is 2, so an alternative way to write the equation of the line is $x=2,\ \frac{y-6}{4}=\frac{z+2}{-3}$.

5 **Make sure that you can identify points on a line correctly** Students often think that if a particular point lies on a line, then scalar multiples of that point also lie on the same line. This is not normally the case (except for lines passing through the origin – can you see why?)

e.g. The point with position vector $\begin{pmatrix}2\\3\\4\end{pmatrix}$ is on the line $\mathbf{r}=\begin{pmatrix}2\\3\\4\end{pmatrix}+\lambda\begin{pmatrix}1\\1\\1\end{pmatrix}$ but the point with position vector $\begin{pmatrix}4\\6\\8\end{pmatrix}$ is not, since the equations $\begin{matrix}4=2+\lambda\\6=3+\lambda\\8=4+\lambda\end{matrix}$ cannot be solved simultaneously, so $\begin{pmatrix}4\\6\\8\end{pmatrix}$ does not satisfy the equation of the line .

6 **Make sure you use different symbols to represent the parameters in the equations of different lines** This must be done in order to avoid confusion. If the same symbol were used it would imply that the parameters in each line always have equal values, which is certainly not true. Different symbols are used for the parameters to indicate that they are separate values.

e.g. Lines l_1 and l_2 have vector equations: $\mathbf{r}=\begin{pmatrix}4\\3\\0\end{pmatrix}+\lambda\begin{pmatrix}5\\2\\3\end{pmatrix}$ and $\mathbf{r}=\begin{pmatrix}-5\\4\\1\end{pmatrix}+\mu\begin{pmatrix}2\\-1\\0\end{pmatrix}$.

7 **Remember that shortest distances always involve perpendicular lines** The shortest distance from a point to a line is the length of a line segment perpendicular to the line, from the point to the line. The shortest distance between two skew lines is the length of a line perpendicular to both lines.

8 **Remember that the distance between two parallel lines is the same as the distance from any point on one line to the other line** You cannot use the formula for the distance between two skew lines to find the distance between two parallel lines, since the vector product of parallel vectors is zero. Instead, remember that the distance between two parallel lines is the distance from any point on one line to the other line, and use the formula for the distance of a point from a line.