Beckfoot	Further Maths	A2 Discrete	Year 13

Graph Theory

1	**Planar**	A graph is planar if it can be drawn in two dimensions without any of the edges crossing.
2	**Subgraph**	A subgraph of a graph is a subset of the vertices together with a subset of the edges.
3	**Subdivision**	A subdivision of a graph occurs when a new vertex is added on an edge so that the edge becomes two edges. So if there is an edge AB, and you add a vertex P so that there is no longer an edge AB, but instead there are two edges AP and BP, then you have subdivided the graph.
4	**Isomorphism**	A one-to-one mapping between the elements of two groups, such that the structures of the two groups are preserved. This means that each element of one group is associated with an element of the other, so that the result of combining two elements in one group maps to the result of combining the equivalent elements in the other group. If a Cayley table is made for each group, with the elements listed in the same order as the elements they are mapped to, then the two Cayley tables will have the same structure.
5	**Cayley Table**	A table showing the results of combining the elements in a set.
6	**You should know the notation for complete graphs** The complete graph with n vertices is denoted as Kn. The complete bipartite graph with m vertices in one set and n in the other is denoted as Km,n. In particular, K5 and K3,3 are important in the use of Kuratowski's theorem.	

Network Flows

1	**Remember the definition of a flow-augmenting path** In particular, remember that a flow-augmenting path may include backward arcs carrying a non-zero flow.
2	**Remember that forward arcs in the minimum cut must be saturated** It is useful to bear this in mind when carrying out the labelling procedure.
3	**Make sure that you can deal with multiple sources and sinks** Remember to add a supersource and supersink in networks with multiple sources and sinks.
4	**Make sure that you can deal with a vertex of restricted capacity** You should replace a vertex of restricted capacity with two vertices connected by an arc with capacity equal to the capacity of the vertex.
5	**Remember how to deal with minimum capacity** If an arc has a minimum capacity, then you must start with an initial feasible flow which satisfies the minimum capacities. The back flow for each arc will then be the amount by which the capacity can be reduced: i.e. the flow – the minimum capacity, and the excess capacity for each arc will be the maximum capacity – the flow. The labelling procedure will then tell you by how much the flow can be increased, so you must add this to the initial flow.
6	**Make sure that you know the definition of the value of a cut when minimum capacities are involved** Add the maximum capacities of arcs which cross the cut in the direction S to T, and then subtract the minimum capacities of arcs which cross the cut in the direction T to S.

Critical Path Analysis

1	**Get plenty of practice with cascade charts** Some students find cascade charts confusing at first. The best way to overcome this is to practise on questions – the Notes and Examples give some examples.
2	**Remember that the area under a resource histogram gives the total amount of that resource required for the project** For example, the area beneath a histogram with number of bulldozers required on the vertical axis and days on the horizontal axis will give the total number of bulldozer-days required for the project, so if you know the cost of hiring a bulldozer and driver for a day, you can work out the total cost of bulldozer hire for the project by multiplying the area under the histogram by the daily hire cost.

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LP – Simplex Tableau

1	**Be careful with your calculations when carrying out pivots** A careless error can make the whole problem go wrong, and it is sometimes quite difficult to spot a minor slip.
2	**Choose your pivot column sensibly** The conventional method is to choose the column with the most negative entry in the objective row. However, if this results in a pivot element of, say, 7, which will mean dividing through by 7 and having to deal with a lot of unpleasant fractions, then it is worth looking at the other possibilities. See what the pivot element would be in each column – a pivot element of 1 is ideal! Similarly, if the [illegible]-values give a choice of pivot elements in a particular problem, choose the easiest option!
3	**Make sure that you understand how to interpret the tableau** Check that you know what is meant by basic and non-basic variables, and that you know their significance in finding a solution from the tableau.

LP – Game Theory

1	**Remember to transform the game to make the pay-offs positive** If you add n to each element, the objective function becomes P –v = -n
2	**Don't forget the constraint for the sum of the probabilities** This is given by p + q + ... ≤1 . You should write it as an inequality even though you know it will be an equality – you should find that the slack variable for this constraint ends up as zero.

Subgroups and Isomorphisms

1	**Look for self-inverse elements when finding subgroups** When looking for subgroups, any element which is its own inverse, together with the identity element, forms a subgroup of order 2.
2	**Remember that a subgroup must include the identity element** It can be tempting, when you spot a block within a table consisting of a closed set of elements, to assume that this is a subgroup. However, if the identity element is not included, then it is not a subgroup.
3	**Make sure you include the trivial subgroup when listing proper subgroups** Don't forget that the trivial subgroup, (the identity element only) is always a subgroup. Unless you are asked for "non-trivial" subgroups, you should include the trivial subgroup. However, the group itself is not a proper subgroup.
4	**Make sure you understand what is meant by an isomorphism** Remember that an isomorphism is a mapping between two groups which preserves the structure of the group.
5	**Remember Lagrange's theorem** Lagrange's theorem, that the order of a subgroup is a factor of the order of the parent group, is useful when looking for subgroups.

Group Theory

1	**Remember that group operations are not necessarily commutative** In some cases group operations may be commutative, but in general work you must make sure that you preserve the correct order when applying an operation.
2	**Make sure that you know the group axioms Learn these carefully.**
3	**Make sure that you prove the group axioms for all cases** If you are asked to prove that a given set with a given operation is a group, you must prove it for all possible cases. For a small, finite group, a Cayley table will allow you to check for closure, inverses and the identity element. However, you must prove associativity in the general case. It is not sufficient to show that it is true in a few cases. For an infinite group, you must prove closure and inverses in the general case.
4	**Make sure you understand what is meant by the period of an element** The period of an element is the smallest power of that element which gives the identity element.
5	**Remember that the identity element has period 1** If you are asked to give the periods of all elements in a group, don't forget to include the identity element, which has period 1.
6	**Make sure you know what is meant by a cyclic group** A cyclic group can be generated by a single element, whose order is equal to the order of the group.