Methods of proof

1	**If you are not sure where to start, try anything**! Draw a diagram if appropriate, or try to state the problem algebraically. Don't worry if you try some approaches which don't seem to work – even the wrong approach often helps you to get a better feel for the problem.
2	**Make sure that each step of your proof is always true** For example, if you are dividing through by an algebraic quantity, could that quantity ever be zero? If so, it invalidates this step of the proof unless you specifically exclude that possibility.
3	**Be careful with your algebra** When you are writing your proof you must take extra care to ensure that the algebra is correct.

Working with radians

1	Where possible, leave your answer in terms of π When an angle is a simple multiple or fraction of 180°, then leave your answers in terms of π .
2	You must know the notation for angles in radians When an angle is given as a multiple or fraction of π you can assume it is in radians. Otherwise you write, say, 0.7 rads. In some older textbooks 0.7 rads might be written as 0.7^c .
3	Make sure that your calculator is in the right mode Remember when you want to work out, say, sin π/3 , make sure your calculator is in 'rad' mode.
4	Make sure that solutions to equations lie in the required range When you are solving a trigonometric equation make sure you check what range the solutions are to lie in and give all the solutions within that range. If the range is given in radians, e.g $0 \leq \theta \leq \pi$, then give your answers in radians. If the range is given in degrees, e.g. $0 \leq \theta \leq 360$, then give your answers in degrees.

Circular measure and small angle approximations

1	Make sure that your calculator is in the right mode Remember when you want to work out, say, sin π/3 , make sure your calculator is in 'rad' mode.
2	Make sure that angles are in radians before using sector formulae The formulae for arc length (rθ) and sector area (½ r^2 θ) can only be used when the angle, θ , is in radians. To change degrees to radians multiply by π/180 .
3	You must know the relationship between the radius and tangent of a circle Remember that the radius of a circle and the tangent to a circle meet at right angles.
4	Remember about the small angle approximations These can be useful to approximate trigonometric functions, so that you can find the approximate solution to an equation. They will be important when you learn about differentiating and integrating trigonometric functions.

Sequences

1	**Know the definitions** Make sure that you know the meaning of all the terms used in this unit, such as sequence, series, increasing sequence, decreasing sequence, arithmetic sequence, geometric sequence, periodic sequence, deductive definition and inductive definition. Check the glossary to make sure.
2	**Be careful to use and interpret the ∑ notation correctly** The numbers above and below the ∑ tell you the first and last term of the sum. So $\sum_3^7 a_k$ means the sum of the 3rd, 4th, 5th, 6th and 7th terms of the sequence a_k, i.e. $\sum_3^7 a_k = a_3 + a_4 + a_5 + a_6 + a_7$

Arithmetic sequences & series

1	**Know the formulae** Make sure that you know the formulae for the kth term of an arithmetic sequence and the sum of the terms of an arithmetic sequence. It's a good idea to make sure you can prove these formulae. This will help you to remember them.
2	**Think about whether your answers are reasonable** It is always a good idea to think carefully about whether your answer is sensible – e.g. if d is negative, the kth term should be smaller than the first term.

Geometric sequences

1	**Know the formulae** Make sure that you know the formulae for the kth term of a geometric sequence, the sum of the first n terms of a geometric sequence, and the sum to infinity of a convergent geometric sequence. It's a good idea to make sure you can prove these formulae. This will help you to remember them.
2	**Beware of simple arithmetical errors** Be careful if the common ratio of a geometric sequence is negative. Make sure that you are using your calculator correctly to find powers of a negative number – use brackets if necessary.

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Maths	A2 Pure - FUNCTIONS	Year 13

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Functions, graphs and transformations

1	**Make sure that you know what all of the terminology means** Check that you know the meaning of all the terminology relating to mappings and functions, and in particular, when a mapping is a function.
2	**Know what effect a transformation has on the equation and graph** Make sure that you know the effect on the equation of a graph of translations, stretches and reflections.
3	**Take care when doing multiple transformations** Be careful when you are using more than one transformation. Sometimes changing the order can give a different result.

The modulus function

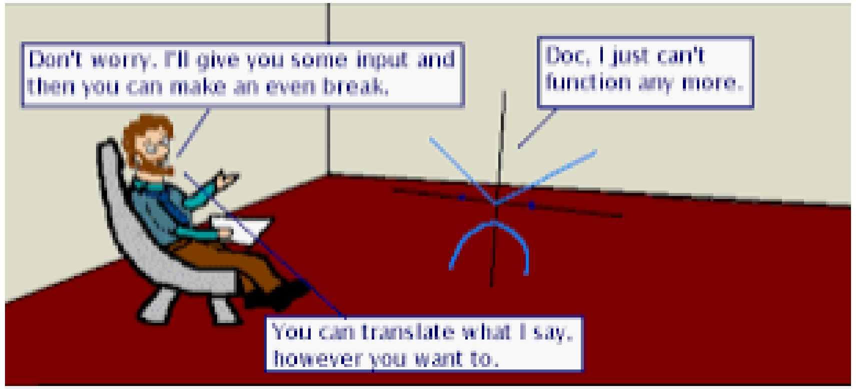
1	**Check that you have the right number of solutions** Be careful when solving equations involving a modulus function that you have the correct number of solutions. Sketching a graph is helpful, and you should also check your solution(s) by substituting back into the original equation.
2	**Take care with inequality signs, especially when they involve negative numbers** When solving inequalities involving a modulus sign, be very careful with the inequality symbol. Remember that you need to reverse it if you are multiplying or dividing through by a negative number. Check your answer by substituting a number from within the solution set into the original inequality.

Composite and inverse functions

1	**For composite functions, make sure you are applying the functions in the right order** Be careful to apply functions in the correct order when finding composite functions. Remember that the function fg means "first apply g, then apply f to the result".
2	**Remember: only a one-to-one function has an inverse function** Sometimes you can define a function with a restricted domain so that it does have an inverse function: for example, $f(x) = x^2$ is a many-to-one function for $x \in \mathbb{R}$, and so does not have an inverse, but if the domain is restricted to $x \geq 0$, then the function is one-to-one and the inverse function $f^{-1}(X) = \sqrt{x}$
3	**When finding the domain or range for f^{-1}, look at the limits of the original function** Notice that the domain of an inverse function f^{-1} is the same as the range of f, and the range of f^{-1} is the same as the domain of f.

The reciprocal trigonometric and inverse trigonometric functions

1	**Make sure you give roots to an equation in the range requested** When solving an equation make sure that you check: • what range the roots should lie in • whether the roots should be given in radians or degrees.
2	**Never cancel a factor in an equation** In an equation such as $\sin\theta - \sin\theta\cos\theta = 0$, never cancel out the term $\sin\theta$ because you will lose the roots to the equation $\sin\theta = 0$. So never cancel – always factorise.
3	**Work from one side of an identity which you are trying to prove** When trying to prove an identity only ever work with one side of the identity. Never try to rearrange it and cancel out terms.



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Maths | A2 Pure - DIFFERENTIATION | Year 13

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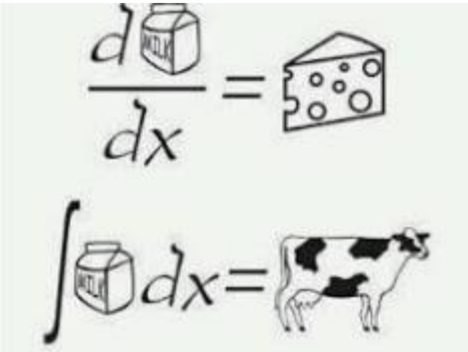
The shape of curves

1 **Remember that points of inflection are not always stationary points** A point of inflection is where the curve changes from convex to concave or vice versa. This may be at a stationary point, but not always.

2 **Be careful when the second derivative is zero** Although the second derivative is always zero at a point of inflection (stationary or non-stationary), the converse is not necessarily true: there may be points where the second derivative is zero that are not points of inflection. At a point of inflection, the second derivative changes sign.



The chain rule

1 **Make sure you remember the du/dx part of the chain rule**

Example: Differentiate y = √(1+2x)

✗ Wrong: $\frac{dy}{dx}=\frac{1}{2}(1+2x)^{-\frac{1}{2}}$

✓ Right: $\frac{dy}{dx}=\frac{1}{2}(1+2x)^{-\frac{1}{2}}\times 2=(1+2x)^{-\frac{1}{2}}$

2 **Recognise situations when the chain rule should be used** The chain rule should be used for functions which can be written in the form y = f(u), where u is a function of x. It cannot be used to differentiate functions which are a product of two functions – this requires the product rule which is covered in the next section.

Example: Find the turning points of $y=\frac{x^2}{1-x}$

First use the quotient rule to differentiate:

$$y=\frac{x^2}{1-x},\Rightarrow\frac{dy}{dx}=\frac{(1-x)2x-x^2(-1)}{(1-x)^2}=\frac{2x-x^2}{(1-x)^2}$$

✗ Wrong: $\frac{dy}{dx}=0$ when $2x-x^2=(1-x)^2$, etc

or $\frac{dy}{dx}=0$ when $2x-x^2=0$ or $(1-x)^2$, etc

✓ Right $\frac{dy}{dx}=0$ when $2x-x^2=0,\Rightarrow x(2-x)=0$

$\Rightarrow x=0,\ y=0,$ or $x=2,\ y=-4$

The product and quotient rules

1 **Make sure you use the product rule correctly**

Example: Differentiate $y=x\sqrt{(1+x)},\ u=x,\ v=\sqrt{(1+x)}$

✗ Wrong $\frac{dy}{dx}=\frac{du}{dx}\times\frac{dv}{dx}=1\times\frac{1}{2}(1+x)^{-\frac{1}{2}}$

✓ Right $\frac{dy}{dx}=u\frac{dv}{dx}+v\frac{du}{dx}=x\times\frac{1}{2}(1+x)^{-\frac{1}{2}}+1\times(1+x)^{\frac{1}{2}}$

2 **Don't mix up the chain rule and the product rule** Functions like y = sin 2x, y = ln(1 + x^2) – use the chain rule Only use the product rule when the function clearly splits up into a product

3 **Make sure you use the quotient rule correctly Don't get 'u' and 'v' mixed up** Remember the negative sign in the numerator

$y=\frac{u}{v}$

✗ Wrong: $\frac{dy}{dx}=\frac{v\frac{du}{dx}+u\frac{dv}{dx}}{v^2}$ $\frac{dy}{dx}=\frac{u\frac{dv}{dx}-v\frac{du}{dx}}{v^2}$

✓ Right $\frac{dy}{dx}=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$

4 **Be careful when finding stationary points of quotient functions** Once you have found dy/dx , to find the stationary point you solve dy/dx = 0. This is true when the numerator of the fraction equals zero, NOT the denominator, OR when numerator = denominator!

SEE EXAMPLE ON THE LEFT

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Maths	A2 Pure – Algebra and Trig	Year 13

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The general binomial expansion

1	**Be careful when working with fractions** It is very easy to make mistakes when finding binomial expansions where n is a fraction, as the binomial coefficients can be quite complicated to work out. Write out the working carefully and check your work.
2	**Be careful with signs** There are often several negative signs involved in each term of a binomial expansion. Use brackets if it helps to make your work clearer, and always check your work.
3	**Know how to deal with cases where the first term in the bracket is not 1** To expand a function of the form $(a + x)^n$ for a $\neq 1$ when n is not a positive integer, take out a factor to give $a^n (1 + x/a)^n$. Remember that when you have expanded $(1 + x/a)^n$ you must then multiply by a^n .
4	**Make sure that you know the validity of your expansion** Remember that $(1 + x)^n$ where n is not a positive integer is valid only for $-1 < x < 1$, and $(1 + ax)^n$ is valid only for $-1/a < x < 1/a$

Rational expressions

1	**Remember that cancelling means dividing, not subtracting** Example: Simplify $\frac{x^2+x}{x}$ WRONG: $\frac{x^2+\cancel{x}}{\cancel{x}} = x^2$ RIGHT: $\frac{x^2+x}{x} = \frac{x(x+1)}{x} = x+1$ x is a factor of both top and bottom
2	**Make sure that you can add, subtract, multiply and divide fractions with confidence** See the examples in the textbook
3	**Remember to check solutions to equations** Always substitute your answer back into the original equation to check (use the fractions key on your calculator so that you are working with exact numbers).

The compound angle formulae

1	**Make sure solutions to an equation are in the right range** When solving an equation make sure that you check: • what range the solutions should lie in • whether the solutions should be in radians or degrees.
2	**Work from one side of an identity which you are trying to prove** When trying to prove an identity only ever work with one side of the identity. Never try to rearrange it and cancel out terms.

Partial fractions

1	**Make sure that you know the correct form for each type of partial fractions** Check that you know the form for a fraction with linear factors and for a fraction with a repeated factor.
2	**Remember to check your answer** You can always check your partial fractions by adding them up and making sure that you get the original fraction.
3	**Think about whether to use substitution or equating coefficients in each case** You can make things very much easier by a sensible choice of value for substitution. Remember that you can use a combination of substitution and equating coefficients – sometimes after one or two substitutions, the other constants can be found very quickly by equating coefficients.
4	**When using partial fractions to find binomial expansions, make sure the expression in each bracket is in the correct form** You can only use the binomial expansion for negative values of n if the expression in the bracket is in the form $1 \pm kx$.
5	**Make sure that you know for what values of x an expansion is valid** You need to write down the values of x for which the expansion is valid for each part of the expansion, and then find the values of x for which both expressions are true.

Alternative forms

1	**Read the question carefully** Check which form of $r\sin(\theta \pm \alpha)$ or $r\cos(\theta \pm \alpha)$ the question is looking for.
2	**Make sure that you give solutions in the correct range** When solving an equation make sure that you check: • what range the solutions should lie in • whether the solutions should be in radians or degrees.

Maths	A2 Pure – FURTHER DIFFERENTIATION	Year 13

Differentiating exponentials and logarithms

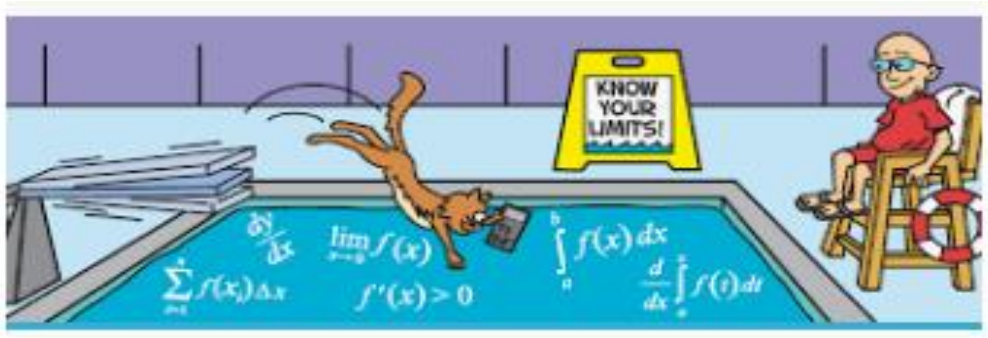
1	**Don't mix up the derivative of x^N with that of e^X** ✗ ***Wrong*** $\frac{dy}{dx} = 2xe^{2x-1}$ ✓ ***Right*** $\frac{dy}{dx} = 2e^{2x}$
2	**Don't get the chain rule and product rule mixed up, especially when differentiating log functions** ✗ ***Wrong*** $\frac{dy}{dx} = 2x\ln + \frac{1}{x}(1+x^2)$ ✓ ***Right*** $\frac{dy}{dx} = \frac{1}{1+x^2} \times 2x = \frac{2x}{1+x^2}$ Note that $\ln(1+x^2)$ is **not** a product of '$\ln$' (which is meaningless) with '$(1+x^2)$'. It is the composite of the two functions $f(x) = \ln x$ and $g(x) = 1+x^2$. So the chain rule is needed to differentiate this.

Differentiating trigonometric functions

1	**Remember that the derivative results rely on measuring x in radians** Example: Differentiate $y = \sin x^\circ$ ✗ ***Wrong*** $\frac{dy}{dx} = \cos x^\circ$ ✓ ***Right*** $y = \sin\frac{\pi}{180}x$, $\frac{dy}{dx} = \frac{\pi}{180}\cos\left(\frac{\pi}{180}x\right) = \frac{\pi}{180}\cos x^\circ$

Implicit differentiation

1	**Make sure that you understand the process of differentiating an equation implicitly** Example: Differentiate implicitly $x^2 + 2y^2 = 10$ ✗ ***Wrong*** $x^2 + 2y^2 = 10$ $\Rightarrow \frac{dy}{dx} = 2x + 4y\frac{dy}{dx}$ $\Rightarrow \frac{dy}{dx}(1-4y) = 2x$ $\Rightarrow \frac{dy}{dx} = \frac{2x}{1-4y}$ ✓ ***Right*** Differentiating the equation implicitly: $2x + 4y\frac{dy}{dx} = 0$ $\Rightarrow \frac{dy}{dx} = -\frac{2x}{4y} = -\frac{x}{2y}$.



Finding areas

1 **Make sure you get x's and y's the right way round, especially when finding an area between a curve and the y-axis** The area between a curve and the y-axis is given by $\int_c^d x\, dy$. You must express x in terms of y so that you can integrate with respect to y. Remember that the limits of the integration are y-values, not x-values.

2 **Draw diagrams** Always sketch the curve (if you are not given a diagram) when you are finding an area, and shade the relevant region.

Integration by parts

1 **Be careful with signs when using the integration by parts formula**

Example Find $\int x\sin x \, dx$

✗ **Wrong** $u = x \quad \frac{dv}{dx} = \sin x \quad \Rightarrow v = -\cos x$

$\int x\sin x\, dx = -x\cos x - \int \cos x\, dx$

$= -x\cos x - \sin x + c$

✓ **Right** $u = x \quad \frac{dv}{dx} = \sin x \quad \Rightarrow v = -\cos x$

$\int x\sin x\, dx = -x\cos x - \int (-\cos x) dx$

$= -x\cos x + \int \cos x\, dx$

$= -x\cos x + \sin x + c$

2 **When using the integration by parts formula, remember to integrate to find 'v' rather than differentiating.**

Example Find $\int xe^{2x} dx$

✗ **Wrong** $\int xe^{2x} dx \; u = x, \frac{dv}{dx} = e^{2x} \Rightarrow v = 2e^{2x}$ (etc)

✓ **Right** $\int xe^{2x} dx \; u = x, \frac{dv}{dx} = e^{2x} \Rightarrow v = \int e^{2x} dx = \frac{1}{2}e^{2x}$ (etc)

Integration by substitution

1 **Remember to substitute for dx in the integral when integrating by substitution**

✗ **Wrong** Let $u = 2x-1 \Rightarrow x = \frac{1}{2}(u+1)$

$\Rightarrow \int x(2x-1)^4 = \int \frac{u+1}{2} \times u^4$

$= \frac{1}{2}\int u^5 + u^4$

$= \frac{1}{12}u^6 + \frac{1}{10}u^5 + c$

✓ **Right** Let $u = 2x-1 \Rightarrow x = \frac{1}{2}(u+1)$

also $\frac{du}{dx} = 2 \Rightarrow dx = \frac{1}{2}du$

$\Rightarrow \int x(2x-1)^4 dx = \int \frac{u+1}{2} u^4 \times \frac{1}{2} du$

$= \frac{1}{4}\int (u^5 + u^4) du$

$= \frac{1}{24}u^6 + \frac{1}{20}u^5 + c$

2 **Remember to change the limits of a definite integral when making a substitution** When you change the variable in an integration (from x to u say) by making a substitution, you must change the limits of the integration from values of x to the equivalent values of u.

3 **Don't mix up the derivatives and integrals of sin x and cos x.** The derivative of sin x is cos x, the integral is -cos x

Example Find $\int \sin x\, dx$

✗ **Wrong** $\int \sin x\, dx = \cos x + c$

✓ **Right** $\int \sin x\, dx = -\cos x + c$

4 **Be careful with signs when substituting values into definite integrals** Example Evaluate $\int_0^{\pi/3} \sin x\, dx$

✗ **Wrong** $\int_0^{\pi/3} \sin x\, dx = [-\cos x]_0^{\pi/3} = -\cos\frac{\pi}{3} + \cos 0 = -\frac{1}{2}$

✓ **Right** $\int_0^{\pi/3} \sin x\, dx = [-\cos x]_0^{\pi/3} = -\cos\frac{\pi}{3} + \cos 0 = -\frac{1}{2} + 1 = \frac{1}{2}$

5 **Make sure that you never integrate across an asymptote when evaluating an integral. EXAMPLE TO THE RIGHT →**

Further techniques for integration

1 **Remember when to use logarithms in integration** Some students make the mistake of wrongly using logarithms when integrating inverse powers of linear functions of x, especially in the context of integration using partial fractions.

Example Find $\int \frac{1}{(x+1)^2} dx$.

✗ **Wrong** $\int \frac{1}{(x+1)^2} dx = \ln(x+1)^2 + c$.

✓ **Right** $\int \frac{1}{(x+1)^2} dx = \int (x+1)^{-2} dx = \frac{(x+1)^{-1}}{-1} + c = -\frac{1}{x+1} + c$.

2 **Make sure you use the correct integration technique when dealing with polynomial fractions** Small changes in the function you are integrating can produce quite different results.

For example: $\int \frac{2x+1}{x^2+x-2} dx = \ln|x^2+x-2| + c$, as the numerator of the fraction is the derivative of the denominator.

However: $\int \frac{3x}{x^2+x-2} dx = \int \frac{3x}{(x+2)(x-1)} dx$

$= \int (\frac{2}{x+2} + \frac{1}{x-1}) dx$

$= 2\ln|x+2| + \ln|x-1| + c$

using partial fractions

Example Find $\int_1^3 \frac{1}{(x-2)^2} dx$

✗ **Wrong** $\int_1^3 \frac{1}{(x-2)^2} dx = [-(x-2)^{-1}]_1^3$

$= -1 - 1$

$= -2$

✓ **Right** The integral is not defined, as it represents an area between $x = 1$ and $x = 3$; but the integrand is not defined when $x = 2$.

Parametric curves

1	**Make sure that you are familiar with the trigonometric identities.** **SEE EXAMPLES IN TEXT BOOK**
2	**Remember that each value of the parameter corresponds to a particular point on the curve** You may be asked to find, say, the equation of a tangent to the curve at the point with parameter t. This will give you an equation in terms of t as well as y and x – many students find this confusing. Each value of t corresponds to a particular point on the curve, and if you substitute that value for t into the tangent, that gives the tangent at that specific point.

Vectors in three dimensions

1	Use vector notation correctly Remember that in handwriting you should underline vectors, or in the case of a vector joining two points, use an arrow above, e.g. $\overrightarrow{AB}$.
2	Make sure you know how to find the resultant of two vectors To find the resultant of two or more vectors simply add them together.
3	Make sure you know how to find the vector joining two points The vector AB is found by $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$
4	Make sure that you know how to find a unit vector To find a unit vector in the same direction as a given vector, a, you divide by the magnitude, IaI

Parametric differentiation and integration

1	**It can help to sketch any curve first on your graphic calculator**
2	**Make sure that you are able to differentiate functions involving sine and cosine confidently** See the work in Further differentiation section 2.
3	**You need to be able to use the chain rule** Make sure that you remember the chain rule:dy/dx=dy/dt x dt/dx. This will help you find the gradient of a curve at a particular point. You can then use it to identify any turning points or find the gradient of the normal. You may find it helpful to use it in this form: $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}, \quad \frac{dx}{dt} \neq 0$
4	**Remember the relationship between the gradients of a tangent and a normal** You will need to use the relationship $m_{\text{normal}} = \frac{-1}{m_{\text{tangent}}}$
5	**Be careful when integrating** Remember that when you are finding the area under a curve defined parametrically, you need to change the variable by using dx/dt , and you need to use the appropriate values of t, not x, as limits of integration.

Maths	A2 Pure – Differential Equations and Numerical Measures	Year 13

Forming and solving differential equations

1	**Make sure you integrate with respect to the correct variable** Example Solve $\frac{dy}{dx} = 2y$ ✗ Wrong $y = \int 2y \, dx = y^2 + c$ ✓ Right $\int \frac{1}{y} dy = \int 2 dx \Rightarrow \ln y = 2x + c$
2	**Remember to add the arbitrary constant after integrating, and be careful not to make algebraic errors in dealing with it** Example Solve $\frac{dy}{dx} = \frac{x^2}{2y}$ ✗ Wrong $\int 2y \, dy = \int x^2 dx$ $\Rightarrow y^2 = \frac{1}{3}x^3 + c$ $\Rightarrow y = \sqrt{\frac{1}{3}x^3} + c$ ✓ Right $\int 2y \, dy = \int x^2 dx$ $\Rightarrow y^2 = \frac{1}{3}x^3 + c$ $\Rightarrow y = \sqrt{\frac{1}{3}x^3 + c}$

The '$+c$' must be inside the square root sign here – the 'green' family of solutions is quite different to the 'red' one above!

Solution of equations

1	**Always sketch a graph** A graph helps to give you some idea of the location of any roots, and also will alert you to possible problems such as discontinuities or repeated roots.
2	**Make sure you state the degree of accuracy of the root** You can give the degree of accuracy by looking at the iterations and seeing how many decimal places have been unchanged for two or three iterations. You can then be reasonably confident that the iteration is correct to that number of decimal places. To be certain, check for a change of sign either side of the root you have found.
3	**Use your calculator efficiently** Using the ANS key repeatedly means that you don't have to keep typing a formula in, and you can generate a number of iterations very quickly. Make sure you know how to do this.
4	**Make sure the root you have found is the one you wanted** Sometimes fixed point iteration converges to a different root to the one you were trying to find. If this happens, try a different starting point, or a different rearrangement. This can also happen with the Newton-Raphson method – if so, try a different starting point.
5	**Use enough decimal places in your working** You need to work with more decimal places than you need in your final answer. The best approach is to store each approximation in your calculator, so that you have maximum accuracy at each stage.

Numerical integration

1	**Make sure you use the formula for the trapezium rule correctly** Remember the 2's in the formula (multiply all the middle numbers by 2 and divide the whole answer by 2 at the end).
2	**Draw a sketch graph** You need to know whether a function is convex or concave, so that you can tell whether the trapezium rule will give an overestimate or an underestimate. Remember that the second derivative can also be helpful.
3	**Be careful with limits when estimating with rectangles** Again, a sketch graph will help. When using areas to estimate integrals, draw a few rectangles at each end of the area, and make sure that you know how many rectangles there should be.